QUANTITATIVE METHODS
FOR ECONOMIC ANALYSIS II

IV Semester

CORE COURSE

BA ECONOMICS

CUCBCSS

(2014 Admission)

UNIVERSITY OF CALICUT

SCHOOL OF DISTANCE EDUCATION

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STUDY MATERIAL

BA – ECONOMICS
(2014 Admission)

QUANTITATIVE METHODS FOR ECONOMIC ANALYSIS II

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Module I

Differential Calculus

Limits and Continuity – Differentiation- Rules, Derivative of single variable and multi variable Functions (except Trigonometric and logarithmic Function), Higher Order Derivatives, Maxima and Minima of Functions. Curvature Properties-Convexity and Concavity. Application of derivatives in economics – Marginal Concepts, Elasticity, Optimisation.

Module 1 – DIFFERENTIAL CALCULUS

Differentiation or differential calculus is mathematical technique that is widely used in economics. In fact, the concept of differentiation provides the foundation of mathematical economics. In mathematics, differentiation comes under the heading calculus. Since the concept of limit is fundamental to the theory of calculus, let us introduce it first.

PART A: LIMITS

1. The Concept of Limit

In economics we have to often find the limiting behaviour of a function as the independent variable approaches a infinite value. For example, we may have to determine the limiting (saturation) level of production as we add more and more units of inputs or we may have to find the limit of sales as sales promotional efforts (like advertising) are increased. To answer such questions, we should understand the concept of limit.

To understand the concept of limit, consider the function *f* denoted by

$$f(x) = \frac{x^2 - 4}{x - 2}$$

Let us assume that x takes a finite value , say x = 2. When x = 2, $f(2) = \frac{0}{0}$

This is an indeterminate solution. However, if we analyze the value of f (x) as x approaches 2 through values both less and greater than 2 (that is, values to the immediate left and right of 2), a different picture emerge. This is shown in the following table.

Table 1: x approaches 2 through values less than 2

x	1.8	1.9	1.99	1.999
f(x)	3.8	3.9	3.99	3.999

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Table 2: x approaching 2 through values greater than 2

x	2.2	2.1	2.01	2.001
f(x)	4.2	4.1	4.01	4.001

From the tables it can be seen that as x approaches 2 from values below or above2, f(x) approaches 4. Then 4 is called the limit of f (x) as x approaches 2 and is written as

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$$

$$\underset{x \to 2}{\text{Lt}} \frac{x^2 - 4}{x - 2} = 4$$

The above expression is read as "limit of $\frac{x^2-4}{x-2}$as x tends to 2 is equal to 4".

In the above function, the values of x approach a finite value (that is 2). But in many cases, the value of x may approach infinity. In that case also the function can have a limiting value. Consider the function $f(x) = 1 - \frac{1}{x}$

Here if we take the value of x = α, we get an indeterminate solution. However as in the above case if analyze the value of f (x) as x approaches α through value both less and greater than α (that is both +ve and –ve values), a different picture emerge. This is shown in the following tables and also in the graph of the function.

Table 3: x approaching α through the +ve values

x	1	2	3	……10	……20	…….50
f(x)	0	0.5	0.7	…...0.9	….0.95	..…0.98

Table 4: x approaches α through –ve values

x	-1	-2	-3	…..-10	…..-20	……-50
f(x)	1	1.5	1.3	…...1.1	….1.05	..…1.02

From the tables it can be seen that as x increases indefinitely, (through either +ve or –ve values), f(x) increases and approaches the limit 1. This can also be seen in the graph of the function (see fig 1). In the figure as x increases through +ve values, the curve rises from left to right and approach the horizontal line drawn at a distance of 1 unit above the ox axis. In the same way as x increases through –ve values, the curve approach unity. In this case it may be said that as x approaches α through the –ve or +ve values, f (x) approaches 1. Thus $\lim_{x \to \alpha} 1 - \frac{1}{x} = 1$

To generalize, given a function y = f (x), if we want to find what value y approaches when x approaches a value (either finite such as 'a' or infinite), we can use the concept of limit. For instance, $\lim_{x \to a} f(x) = l$ implies that the function approaches 'l' as the variable x approaches a finite value 'a'. It may be remembered that, when we discuss the limit of a function as x → a, we are not interested in what happens exactly at x = a. We are only interested in what happens to the function when x assumes values to the immediate left and right of 'a'. This information is shown in figure 2. It cab be seen that the graph of the function approaches 'l' as x approaches 'a'. Similarly, $\lim_{x \to \alpha} f(x) = l$implies that the function approaches 'l' as the variable approaches infinitely.

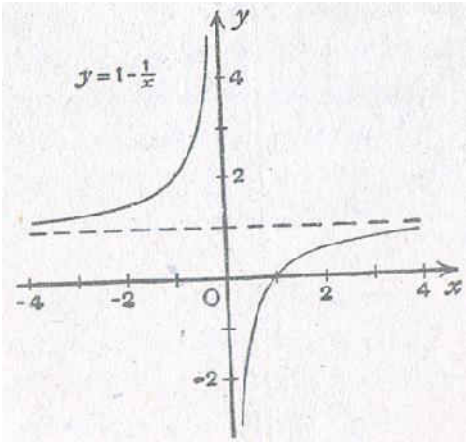


Figure 1

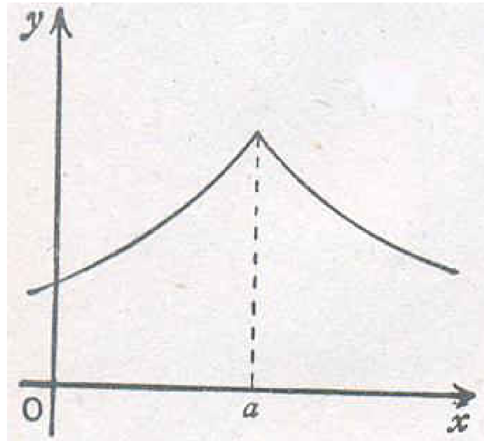


Figure 2

Operation of the limit.

There are certain basic rules, which can help us to find limit.

Rule 1.

$\lim \mathrm{K} =_{x\to a} K$ (where K is a constant).

Eg. $\lim_{x\to 6} 5 = 5$

Rule 2.

$\lim_{x\to a} x^n = a^n$ (where n = any +ve integer).

Eg. $\lim_{x\to 2} x^2 = 2^2 = 4$

Rule 3.

$\lim_{x\to a} k\, f(x) = K \lim_{x\to a} f(x)$ (where K is a constant).

Eg. $\lim_{x\to 2} 5x^3 = 5 \lim_{x\to 2} x^3 = 5(2^3) = 40$

Rule 4.

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

Eg. 1:$\lim_{x\to 2}(x^3 + 4x)$

$$\lim_{x\to 2}(x^3 + 4x) = \lim_{x\to 2} x^3 + 4\lim_{x\to 2} x = 2^3 + 4(2) = 16$$

Eg. 2: $\lim_{x\to 2}(x^3 - 4x)$

$$\lim_{x\to 2}(x^3 - 4x) = \lim_{x\to 2} x^3 - 4\lim_{x\to 2} x = 2^3 - 4(2) = 0$$

Rule 5.

$\lim_{x\to a}[f(x) \times g(x)] = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x)$

Eg:

$$\lim_{x\to 4}[8x \times 6x] = \lim_{x\to 4} 8x \times \lim_{x\to 4} 6x = (8 \times 4) \times (6 \times 4) = 768$$

Rule 6.

$\lim_{x\to a}[f(x) \div g(x)] = \lim_{x\to a} f(x) \div \lim_{x\to a} g(x)$

(Provided$\lim_{x\to a} g(x) \neq 0$)

Eg:

$$\lim_{x\to 2}[8x \div 4x] = \lim_{x\to 2} 8x \div \lim_{x\to 2} 4x = 8(2) \div 4(2) = 16 \div 8 = 2$$

Rule 7.

$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$(Provided $n > 0$)

Eg: $\lim_{x\to 2}(6x^3)^{1/2}$

$= \lim_{x\to 2}(6(2)^3)^{1/2} = 48^{1/2} = \sqrt{48} = 6.93$

To sum up, the logic underlying these rules is to substitute x by the given value to which x is tending.

Other examples

I. Find limit of the following functions.

1. $\lim_{x\to 1} 3 = 3$

2. $\lim_{x\to 6} 2x = 12$

3. $\lim_{x\to 1} 3x^3 - 2x^2 + x + 4$

$= \lim_{x\to 1} 3x^3 - \lim_{x\to 1} 2x^2 + \lim_{x\to 1} x + \lim_{x\to 1} 4$= 3−2 +1+4 = 6

4. $\lim_{x\to 2} \frac{x^5-2}{x^2-3}$

$$= \frac{\lim_{x\to 2} x^5 - 2}{\lim_{x\to 2} x^2 - 3}$$
$$= \frac{32-2}{4-3} = 30$$

5. $\lim_{x\to 2} \sqrt{6x^3+1}$

$$= \lim_{x\to 2}(6x^3+1)^{1/2}$$
$$= [6(2)^3+1]^{1/2}$$
$$= 49^{1/2} = \sqrt{49 = 7}$$

6. $\lim_{x\to 3} \frac{x^2-9}{x-3}$

Here if we simply substitute x =3, as we did in the above examples, the numerator and the denominator becomes zero (i.e. f (4) =0/0) and the solution is indeterminate. However, in many cases the indeterminate solution emerges due to common factors in the numerator and denominator. We may be able to get a determinate solution either by removing these common factors or by dividing the numerator and denominator by the same quantities.

Thus in this case, if we factorize the numerator, we get

$$\lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}\frac{(x+3)(x-3)}{(x-3)}$$
$$= \lim_{x\to 3} x+3 = 3+3 = 6$$

7. $\lim_{x\to 2}\frac{x^2-3x+2}{x-2}$

   Since f(2)=0/0 is indeterminate, we have to factorize

$$\lim_{x\to 2}\frac{(x-1)(x-2)}{x-2}$$
$$= \lim_{x\to 2} x-1 = 1$$

8. $\lim_{x\to 2}\frac{x^2-5x+6}{x-2}$

   Since f (2) = 0/0 is indeterminate, we have to factorize.

$$= \lim_{x\to 2}\frac{(x-3)(x-2)}{x-2}$$
$$= \lim_{x\to 2} x-3 = -1$$

9. $\lim_{x\to 0}\frac{\sqrt{(x+1)}-1}{x}$

   Since f(0) is indeterminate, we have to factorize

$$= \lim_{x\to 0}\frac{[\sqrt{(x+1)}-1][\sqrt{(x+1)}+1]}{x[\sqrt{(x+1)}+1]}$$
$$= \lim_{x\to 0}\frac{\left(\sqrt{(x+1)}\right)^2-1^2}{x(\sqrt{x+1}+1)}$$
$$= \lim_{x\to 0}\frac{(x+1)-1}{x(\sqrt{x+1}+1)}$$
$$= \lim_{x\to 0}\frac{x+1-1}{x[\sqrt{x+1}+1]}$$
$$= \lim_{x\to 0}\frac{x}{x[\sqrt{x+1}+1]}$$
$$= \lim_{x\to 0}\frac{1}{\sqrt{x+1}+1}$$
$$= \frac{1}{1+1} = \frac{1}{2}$$

10. $\lim_{x\to \alpha}\frac{5x^2+3x+15}{x^2+5}$

   Since f(α) is indeterminate, we have to factorize

$$= \lim_{x\to \alpha}\frac{x^2\left(5+\frac{3}{x}+\frac{15}{x^2}\right)}{x^2(1+\frac{5}{x^2})}$$

$$= \lim_{x\to \alpha}\frac{5+\frac{3}{x}+\frac{15}{x^2}}{1+\frac{5}{x^2}}$$

Since α can also be expressed as 1/0, substitute x→ 1/0 in the above equation and it can be seen that as $x \to \frac{1}{0}, \frac{3}{x}, \frac{15}{x^2}$ *and* $\frac{5}{x^2} \to 0$

$$= \frac{5}{1} = 5$$

2. The Continuity of Functions

A function can be either continuous or discontinuous. Using ideas of functions and limits, we have to distinguish between these two. In the previous section when we discussed the limit of a function as x→a, we stated that we are not interested in what happens exactly at x = a [that is f(a)], but only in what happens in the neighbourhood of ‘a’ [that is $\lim_{x\to a} f(x)$]. But in this section, when we discuss the continuity of a function, we are interested in both of these. To define continuity of a function precisely, we have to consider continuity of the function at a particular point in the domain, say at ‘a’. Then, a function y = f(x) is said to be continuous at x = a if two conditions are satisfied, (1) f(a) exists and is finite and (2) $\lim_{x\to a} f(x) = f(a)$

The concept of continuity can be made clear through the following 2 examples.

Example 1. Consider the function y = 3x + 7. Assume that x takes the value 2. When x = 2, y or f(2) = 13. Here f(2) exists and is finite and hence first condition for continuity is satisfied. Now we have to see whether the second condition id satisfied. For this we have to find $\lim_{x\to 2} 3x + 7$. We know that $\lim_{x\to 2} 3x + 7 = 13$. This means $f(2) = \lim_{x\to 2} 3x + 7$and hence the second condition for continuity is also satisfied. We can conclude that the function y = 3x +7 is continuous at x = 2.

Example 2. Consider the function $= \frac{1}{x^3-8}$. When x = 2, y = 1/0 which is undefined. Here f(2) is neither defined nor finite. As the first condition for continuity is not satisfied, it is clear that the function is not continuous. Hence we may conclude that the function $y = \frac{1}{x^3-8}$ is not continuous at x = 2.

Hope the concept of continuity is clear to you. The continuity of a function at a given value can also be analyzed from the graph of the function. There are two conditions that should be satisfied by the graph of a continuous function. (1) There can be no gap in the curve. (2) There can be no jump in the curve.

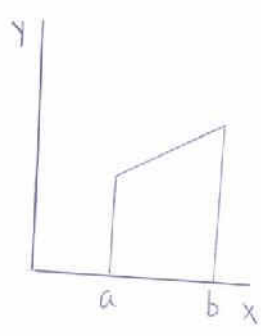


Fig 3

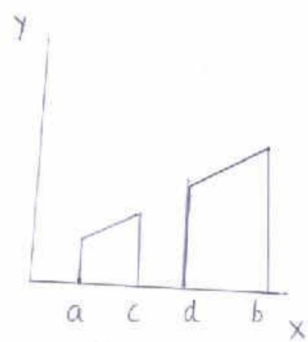


Fig 4

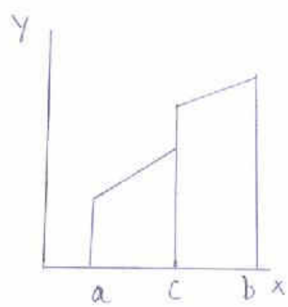


Fig 5

In the above graphs, fig 3 alone represents a continuous function at x = a. Fig 4 is not continuous because there is a gap in the graph and fig 5 is not continuous because there is a jump in the graph. To conclude, loosely speaking a continuous function of one whose graph can be plotted without taking the pencil from the paper.

Examples

I. Discuss the continuity of the following functions.

1. $f(x) = \frac{1}{x}$ $at\ x = 0$

$$f(0) = \frac{1}{0} = \alpha$$

Since f (0) is undefined and not finite, the first condition is not satisfied and the function is not continuous at x = 0.

2. $f(x) = \frac{1}{x}$ $at\ x = 1$

$f(1) = 1$ First condition for continuity is satisfied.

$$\lim_{x \to 1} \frac{1}{x} = 1$$

So we see that

$f(1) = \lim_{x \to 1} \frac{1}{x}$. Hence the second condition for continuity is also satisfied.

So we conclude that the function $f(x) = \frac{1}{x}$ $is\ continous\ at\ x = 1$

II

1. Examine whether the function $y = \frac{1-x^2}{1-x}$is a continuous

Here the value of x at which continuity is to be verified is not given. So we have to take any value. Let x = 1.

$$y = \frac{1-1^2}{1-1} = \frac{0}{0}$$

Since the first condition is not satisfied, the function is not continuous at x = 1.

2. Examine the continuity of the function y = x (x + 1) for x > 0.

At x = 1

f(1) = 2, so first condition satisfied.

$\lim_{x \to 1} x(x+1) = 2$, and the second condition also satisfied.

Hence the function is continuous at x = 1.

Let us check continuity for one more value, at x = 2, f(2) = 6.

$$\lim_{x \to 2} x(x+1) = 6 = f(2)$$

Since both the conditions are satisfied, the function is continuous at x = 2 also.

Exercise

Evaluate the following.

(1) $\lim_{x \to 2} \frac{x^2-3x+2}{x-2}$

Answer: 1

(2) $\lim_{x \to 1} \frac{1-x}{1-\sqrt{x}}$

Answer: 2

(3) $\lim_{x\to 0} \frac{\sqrt{(x+1)-1}}{x}$

Answer: 1/2

(4) $\lim_{x\to 1} \frac{x^2-1}{x-1}$

Answer: 2

(5) $\lim_{x\to 2} \frac{3x^2+2}{x^2-3}$

Answer: 14

(6)$\lim_{x\to 3} \frac{x^2-x-6}{x^2-9}$

Answer: 5/6

(7) $\lim_{x\to -1} \frac{x^5+1}{x+1}$

Answer: 5

(8) $\lim_{x\to 0} \frac{\sqrt{x+3}-\sqrt{3}}{x}$

Answer: $1/2\sqrt{3}$

(9) $\lim_{x\to \propto} \frac{x^3+3x^2+2}{4x^3+2x+3}$

Answer: 1/4

(10) $\lim_{x\to -2} \frac{(x+4)(2x+1)}{x^2+5x+2}$

Answer: 1.5

(11) $\lim_{x\to 1} \frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$

Answer: ¼

(12) $\lim_{x\to 2} \frac{(x^2-5x+6)(x^2-3x+2)}{x^3-3x^2+4}$

Answer: -1/3

PART B: DIFFERENTIATION

In economics we often come across the situation where we want to find change in the dependent variable as a result of a small change in an independent variable. For example for a demand function Q = f (p), we are interested to knowing what happens to quantity Q (the dependent variable) as a result of a small change in the price P (the independent variable). Similarly a given production function, we interested in knowing what happens to output as a result of a small change in the inputs used. Differentiation is a technique, which can be sued when such problems are to be solved. Differentiation is concerned with finding the rate of change in a dependent variable when there is a small (infinitesimally small) change in the independent variable. Let us first see the concept of derivative, which can be developed using the idea of finite introduced in the previous section.

The Derivative.

To express the rate of change in any function, the concept of derivative can be used. The concept of a derivative can be made clear through the following two examples.

Assume that we have a product y (say rice) produced by an input x (say labour). Further assume that as we increase the number of labour (x) by one unit, the amount of rice (y) increases by four units. This relationship can be shown by the function.

$y = 4x$ …. (1)

Now assume that we want to find the increase in rice production (y) when there is a very small increase in quantity of labour (x) used in production process. Let the change in x be Δx and change in y by Δy. Then equation (1) can be written as

$y + \Delta y = 4 (x + \Delta x)$

$y + \mathbf{y =} 4x + 4\Delta x$

$\Delta y = -y + 4x + 4\Delta x$

$\Delta y = -4x + 4x + 4\Delta x$ (from equation 1)

$\Delta y = 4 \Delta x$.

Dividing both R.H.S and L.H.S by Δx, we get.

$$\frac{\Delta y}{\Delta x} = \frac{4\Delta x}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 4 \quad \text{- - - (2).}$$

Note that equation (2) implies that for a small unit change in x (labor) production of y (rice) increases by 4k.g. If we call y of equation (1) the dependent variable and x the independent variable, then the quotient $\Delta y/\Delta x$ shows the rate of change of the dependent variable y with respect to the independent variable x. In other words, $\Delta y/\Delta x$ shows the change of y with respect to a small unit change of x.

Before we expand this further, let us see one more function. Assume that the relation between labor (x) and rice (y) is $y = 4x^2$ ….(3)

Suppose we are interested in knowing the change in y when there is a small increase in x. Assume that when x is increased by a small unit Δx, y increased by Δy and we have

$y + y = 4(x +\Delta x)^2$

$y + \Delta y = 4 (x^2 + 2x\Delta x + \Delta x^2)$

$y + y = 4x^2 + 8x \Delta x + 4\Delta x^2$

$\Delta y = 4x^2 + 8x\Delta x + 4\Delta x^2 – y$

$\Delta y = 4x^2 + 8x\Delta x + 4\Delta x^2 - 4x2$ (From equation 3)
$\Delta \mathbf{y} = 8x\Delta x + 4\Delta x^2$
Dividing both R.H.S and L.H.S by Δx

$$\frac{\Delta y}{\Delta x} = \frac{8x\,\Delta x}{\Delta x} + \frac{4\Delta x^2}{\Delta x}$$
$$\frac{\Delta y}{\Delta x} = 8x + 4\Delta x$$

Since Δx represents an infinitely small change, we may write that $\Delta x \to 0$. Then, applying limit to equation 2, we get

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} 8x + \lim_{\Delta x \to 0} 4\Delta x$$
$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 8x$$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$may be called differential coefficient of y with respect to x or the derivative of y with respect to x. This can be written in a more formal terminology as $\frac{dy}{dx}$. (loosely speaking *dy* stands for Δy and dx stands forΔx.)
Thus we may rewrite the above equation as

$$\frac{dy}{dx} = 8x \quad \ldots (3)$$

The $\frac{dy}{dx}$ is interpreted as the change in y due to a very small unit change in x or we may say that it is the rate of change of y with respect to x.

Thus differentiation is the process of obtaining the derivative of a function. To generalize, for a given function y = f (x), dy/dx shows the change in the dependent variable y due to a change in the dependent variable x. The derivative dy/dx can also be represented as y^I or $\frac{df(x)}{dx}$ or $f^I(x)$ etc.

The method we followed in the above examples to find derivative is round about method. The derivative can be found out directly from a function by applying a few basic rules of formula to a given function. These are listed below.

Rules of Differentiation

<u>Rule 1: Differentiation of a Power Function</u>

$$If\ y = x^n,\ then, \qquad \frac{dy}{dx} = \frac{dx^n}{dx} = nx^{n-1}$$

See the following examples.

Eg. Find the derivative of the following functions using Rule 1

1. $y = x^n$

Applying rule 1 we get

$$\frac{dy}{dx} = 2x^{2-1} = 2x$$

2. $y = x^6$

Applying rule 1 we get

$$\frac{dy}{dx} = 6x^{6-1} = 6x^5$$

3. $y = x$

Applying rule 1 we get

$$\frac{dy}{dx} = 1x^{1-1} = x^0 = 1 \quad (remember\ that\ any\ number\ to\ the\ power\ 0\ is\ 1)$$

Rule 2: Derivative of a sum / difference

If we are given a function y = u + v, where u = f(x) and v = g(x) are functions of x, then the derivative is found using the formula

$$\frac{dy}{dx} = \frac{d^u}{dx} + \frac{d^v}{dx} \quad or\ simply\ \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

That means differentiate each term separately and then add (or subtract).

See the following examples.

Eg. Find the derivative of the following functions using Rule 2

1. $y = x^6 + x^9$

Applying rule 2 we get

$$\frac{dy}{dx} = \frac{dx^6}{dx} + \frac{dx^9}{dx}$$

$$= 6x^{6-1} + 9x^{9-1}$$

$$= 6x^5 + 9x^8$$

2. $y = x^8 + x^3$

Applying rule 2 we get

$$\frac{dy}{dx} = \frac{dx^8}{dx} + \frac{dx^3}{dx}$$

$$= 8x^{8-1} + 3x^{3-1}$$

$$= 8x^7 + 3x^2$$

Similarly if y = u – v, where u = f(x) and v = g(x) are functions of x, then

$$\frac{dy}{dx} = \frac{d^u}{dx} - \frac{d^v}{dx} \quad or\ simply\ \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

3. $y = x^6 - x^9$

Applying rule 2 we get

$$\frac{dy}{dx} = \frac{dx^6}{dx} - \frac{dx^9}{dx} = 6x^5 - 9x^8$$

4. $y = x^8 - x^3$

Applying rule 2 we get

$$\frac{dy}{dx} = \frac{dx^8}{dx} - \frac{dx^3}{dx} = 8x^7 - 3x^2$$

Rule 3: Derivative of a constant

If y = K, where K is a constant, $\frac{dy}{dx} = \frac{dK}{dx} = 0$

Thus, note that, the derivative of a constant is zero.

Eg. Find the derivative of the following functions using Rule 3

Eg. 1. y = 2.

$$\frac{dy}{dx} = \frac{d(2)}{dx} = 0$$

Eg. 2. y = 8.

$$\frac{dy}{dx} = \frac{d(8)}{dx} = 0$$

Rule 4: Derivative of a function with multiplicative constant / additive constant

First consider a function with a multiplicative constant

y = ku

where u is a function of x and it is multiplied by a constant k.

$$Then\ \frac{dy}{dx} = k\ \frac{du}{dx}$$

Thus, when we have to find the derivative of a function in which a variable is multiplied by a constant, keep the constant out and differentiate the remaining.

Eg. Find the derivative of the following functions using Rule 4

1. $y = kx^2$

$$\frac{dy}{dx} = k\frac{dx^2}{dx} = k(2x)$$

2. $y = 5x^6$

$$\frac{dy}{dx} = 5\frac{dx^6}{dx} = 5(6x^{6-1}) = 5(6x^5) = 30x^5$$

3. $y = ax^b$

$$\frac{dy}{dx} = a\frac{dx^b}{dx} = abx^{b-1}$$

Now consider a function with an additive constant, $y = k + u$, where a constant k is added to u, which is a function of x. Then

$$\frac{dy}{dx} = \frac{dk}{dx} + \frac{du}{dx}$$

$$= 0 + \frac{du}{dx} = \frac{du}{dx}$$

Eg. 1. $y = 8 + x^5$

$$\frac{dy}{dx} = \frac{d8}{dx} + \frac{dx^5}{dx} = 0 + 5x^4 = 5x^4$$

Eg. 2. $y = 8 - x^5$

$$\frac{dy}{dx} = \frac{d8}{dx} - \frac{dx^5}{dx} = 0 - 5x^4 = -5x^4$$

Eg. 3. $y = 2x^6 + 7$

$$\frac{dy}{dx} = 2\frac{dx^6}{dx} - \frac{d7}{dx} = 12x^5 + 0 = 12x^5$$

Rule 5: Derivative of a product

Let y = f(x).g(x), where f(x) and g(x) are both differentiable functions. To find its derivative, let u = f(x) and v=g(x)

Then y = uv

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Or it can be written as

$$\frac{dy}{dx} = uv^I + vu^I$$

Thus the derivative of a product is equal to the first function multiplied by the derivative of the second plus the second function multiplied by the derivative of the first.

Eg. Find the derivative of the following functions using rule 5

1. $y = (3x^2)(2x^2)$

Let u = $3x^2$ and v= $2x^2$

$$u^I = \frac{du}{dx} = 6x$$

$$v^I = \frac{dv}{dx} = 4x$$

Substitute in the formula

$$\frac{dy}{dx} = uv^I + vu^I$$

$$\frac{dy}{dx} = (3x^2)(4x) + (2x^2)(6x)$$

$$= 12x^3 + 12x^3 = 24x^3$$

2. $y = 2x^2(5x + 3)$

Let u = $2x^2$ and v= 5x+3

$$u^I = \frac{du}{dx} = 4x$$

$$v^I = \frac{dv}{dx} = 5$$

Substitute in the formula

$$\frac{dy}{dx} = uv^I + vu^I$$

$$\frac{dy}{dx} = (2x^2)(5) + (5x + 3)(4x)$$

$$= 10x^2 + 20x^2 + 12x = 30x^2 + 12x$$

3. $y = (6x^2 + 5)(x^3 + 8x)$

Let u = $6x^2 + 5$ and v = $x^3 + 8x$

$$u^I = \frac{du}{dx} = 12x$$

$$v^I = \frac{dv}{dx} = 3x^2 + 8$$

Substitute in the formula

$$\frac{dy}{dx} = uv^I + vu^I$$

$$\frac{dy}{dx} = (6x^2 + 5)(3x^2 + 8) + (x^3 + 8x)(12x)$$

$$= 18x^4 + 15x^2 + 48x^2 + 40 + 12x^4 + 96x^2$$

$$= 159x^2 + 30x^4 + 40$$

Rule 6: Derivative of a quotient

$$Let\ y = \frac{f(x)}{g(x)}$$

Where f(x) and g(x) are both differentiable functions and g(x)≠0. To find its derivative, let u = f(x) and v = g(x).

$$Then\ y = \frac{u}{v}$$

Its derivative is found using the formula

$$y = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Or it can be written as

$$y = \frac{vu^I - uv^I}{v^2}$$

Thus the derivative of quotient is equal to the denominator multiplied by the derivative of the numerator, minus, the numerator multiplied by the derivative of the denominator, whole divided by the denominator squared.

Eg. Find the derivative of the following functions using rule 6

1. $y = \frac{2x^2+3}{x}$

$$Let\ u = 2x^2 + 3\ and\ v = x$$

$$u^I = \frac{du}{dx} = 4x \quad and \quad v^I = \frac{dv}{dx} = 1$$

Substitute in the formula

$$y = \frac{vu^I - uv^I}{v^2}$$

$$y = \frac{x(4x) - (2x^2 + 3)(1)}{x^2}$$

$$= \frac{4x^2 - 2x^2 - 3}{x^2} = \frac{2x^2 - 3}{x^2}$$

2. $y = \frac{10x^8 - 6x^7}{2x}$

$$Let\ u = 10x^8 - 6x^7\ and\ v = 2x$$

$$u^I = \frac{du}{dx} = 80x^7 - 42x^6 \quad and \quad v^I = \frac{dv}{dx} = 2$$

Substitute in the formula

$$y = \frac{vu' - uv'}{v^2}$$

$$y = \frac{(2x)(80x^7 - 42x^6) - (10x^8 - 6x^7)(2)}{(2x)^2}$$

$$= \frac{160x^8 - 84x^7 - 20x^8 + 12x^7}{4x^2} = \frac{140x^8 - 72x^7}{4x^2} = 35x^6 - 18x^5$$

Rule 7. Differentiation of function - the chain rule.

This is one of the most useful rules of differentiation in economics analysis. The rule can be stated as follows. If we have a function z = f(y), where y is in turn a function of another variable x, say y = g(x), then the derivative of z with respect to x is equal to the derivative of z with respect to y, multiplied by the derivative of y with respect to x.

symbolically $\frac{dz}{dx} = \frac{dz}{dy} . \frac{dy}{dx}$.

Since students usually find some difficulty in understanding this rule, let us illustrate the same with the aid of a simple example. Let x be land, y be wheat and z be bread. Assume that for every unit of x (land), we can produce 3 units of y (wheat). Then we can write y = 3x. Further assume that for every unit of y (wheat), we can produce 10 units of z (bread). Then we can write z = 10y. Now suppose that we want to find the change in bread (z) when there is one unit in land (x). This can be found out by substituting the value of y and also assuming x = 1.

Z = 10y

Z = 10(3x)   since y = 3x

Z = 30 x.

Z = 30.   Since x = 1

Now we can interpret the same using derivative. We know that dy/dx gives us the change in wheat (y) as a result of a small change in land(x). Then we have

$$\frac{dy}{dx} = \frac{d3x}{dx} = 3$$

which implies that the change in wheat (y) as a result of a one unit change in land (x) is 3. Now to find the change in bread (z) due to a small unit change in wheat, we have to find $\frac{dz}{dx}$

$$\frac{dz}{dx} = \frac{d\ 10y}{dx} = 10$$

Now to get the change in bread (z) due to a change in land (x), we have to find $\frac{dz}{dx}$ which can be obtained by multiplying the above two.

Then we get

$$\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{dx} = 10 \times 3 = 30$$

Eg. Find the derivative of the following functions using rule 7

1. $Z = 3y^2 \ where \ y = 2x + 5$

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

$$\frac{dz}{dx} = 6y \quad and \ \frac{dy}{dx} = 2$$

$$\frac{dz}{dx} = (6y)(2) = 12y$$

Substitute y = 2x+ 5, the we get

$$\frac{dy}{dx} = 12(2x + 5)$$

2. $z = \ y^4 + 3y^3 \quad and \ y = x^2$

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

$$\frac{dz}{dx} = 4y^3 + 9y^2 \quad and \quad \frac{dy}{dx} = 2x$$

$$\frac{dz}{dx} = (4y^3 + 9y^2)(2x)$$

$$= 8xy^3 + 18xy^2$$

Substitute y = x^2

$$\frac{dz}{dx} = [4(x^2)^3 + 9(x^2)^2][2x]$$

$$= [4x^6 + 9x^4]2x$$

$$= 8x^7 + 18x^5$$

3. $y = (3x^4 + 5)^6$

$$Let\ u = 3x^4 + 5, then\ y = u^6$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$\frac{dy}{dx} = 6u^5 \quad and \quad \frac{dy}{dx} = 12x^3$$

$$\frac{dy}{dx} = (6u^5)(12x^3)$$

$$= 72x^3u^5$$

$$Substituting\ u = 3x^4 + 5$$

$$\frac{dy}{dx} = 72x^3(3x^4 + 5)^5$$

4. $y = -3(x^2 - 8x + 7)^4$

$$Let\ x^2 - 8x + 7 = u, then\ y = -3u^4$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$\frac{dy}{du} = -12u^3, \quad \frac{du}{dx} = 2x - 8$$

$$\frac{dy}{dx} = -12u^3(2x-8)$$

$$(-24x+96)u^3$$

Substituting u we get

$$\frac{dy}{dx} = (-24x+96)(x^2-8x+7)^3$$

5. $If\ y = 2w^2 + 2, w = 6z^2,\ z = 4x + x^2, find\ \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy}{dw} \times \frac{dw}{dz} \times \frac{dz}{dx}$$

$$\frac{dy}{dw} = 4w, \qquad \frac{dw}{dz} = 12z, \qquad \frac{dz}{dx} = 4+2x$$

Substituting we get

$$\frac{dy}{dx} = (4w)(12z)\ \ (4+2x)$$

$$= (48w^2)(4+2x)$$

Substituting w, we get

$$= [48(6z^2z)][4+2x]$$

$$= [288z^3][4+2x]$$

Substituting Z, we get

$$= [288(4x+x^2)^3][4+2x]$$

$$= [288[x(4+x)]^3][4+2x]$$

$$\frac{dy}{dx} = 288x^3(4+x)^3\ (4+2x)$$

The chain rule that we have just seen is one that is widely used. However, at many instances we can use a short cut method called 'The generalised power function rule', which is derived from the chain rule itself to arrive at the result quickly.

Rule 8: The Generalized power function Rule

Let us illustrate this rule through an example. Consider the function y = (x + 3) ^{6}. By chain rule we let u = x + 3 and y = u6. Then.

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 6u^5 = 6(x+3)^5$$

The same result can be obtained directly and easily using the generalized power function rule. By this rule we have to first take derivative of (x + 3)6 by rule 1 and then multiply it by the derivative of x + 3. That is

$$y^{|} = \frac{d(x+3)^6}{dx} \times \frac{d(x+3)}{dx}$$

$$y^{|} = 6(x+3)^5 \times 1 = 6(x+3)^5$$

We may generalize the above to state the rule asfollows.

Given y = $[g(x)]^{n,}$ where g(x) is a differentiable function and n is any real number,

$$y^{|} = n[g(x)]^{n-1} \times g^{|}$$

where $g^{'}$ = the derivative of g(x).

Eg. Find the derivative of the following functions using rule 8

1. $y = (6x+8)^4$

Applying the rule

$$y^{|} = \frac{d[(x^5+7)]^6}{dx} \frac{d[x^5+7]}{dx}$$

$$y^{|} = 6(x^5+7)^5 \times 5x^4$$

$$= 30x^4(x^5+7)^5$$

2. $y = (6x+8)^4$

Applying the rule

$$y^{|} = 4(6x+8)^3 \times 6 = 24(6x+8)^3$$

3. $y = \frac{1}{2x^2+10x+9}$

This can be written as

$$y = (2x^2+10x+9)^{-1}$$

Applying the rule

$$y^{|} = -1(2x^2+10x+9)^{-1-1}(4x+10)$$

$$y^{|} = -1(2x^2+10x+9)^{-2}(4x+10)$$

$$y^{|} = -(4x+10)(2x^2+10x+9)^{-2}$$

$$y^{|} = \frac{-(4x+10)}{(2x^2+10x+9)^2}$$

4. $y = \sqrt{6x}$

This can be written as

$$y = (6x)^{1/2}$$

Applying the rule

$$y^{|} = \frac{1}{2}(6x)^{1/2-1} \times 6$$

$$y^{|} = \frac{1}{2}(6x)^{-1/2} \times 6$$

$$y^{|} = 3(6x)^{-1/2}$$

$$y^{|} = \frac{3}{(6x)^{1/2}} = \frac{3}{\sqrt{6x}}$$

5. $y = \frac{1}{\sqrt{6x}}$

This can be written as

$$y = (6x)^{-1/2}$$

Applying the rule

$$y^{|} = \frac{-1}{2}(6x)^{-1/2-1} \times 6$$

$$y^{|} = \frac{-1}{2}(6x)^{-3/2} \times 6$$

$$y^{|} = -3(6x)^{-3/2}$$

$$y^{|} = \frac{-3}{(6x)^{3/2}}$$

$$y^{|} = \frac{-3}{\sqrt{(6x)^3}}$$

Rule 9: Differentiation of an implicit function

In economics we often come across explicit functions in which the dependent variable is at the LHS and the independent variable at the RHS of the equal sign. However we may also have to deal with implicit functions where both variables are on the same side of the equal sign. For example y – 3x = 0 is an implicit function and y = 3x is an explicit function. As in this example, some implicit functions can be easily converted to explicit functions by solving for the independent variable and their differentiation can be easily done. However there are certain implicit functions that cannot be easily converted to explicit functions and hence their differentiation requires special rules for example $8x^4 - 2y^5 = 56$. For this we use implicit differentiation rule.

Consider an implicit function 4x – 5y = 10. This can be differentiated by implicit differentiation by the following two steps.

Step 1: The first step is to differentiate both sides of the equation with respect to x while treating y as a function of x. thus

$$\frac{d(4x)}{dx} - \frac{d(5y)}{dx} = \frac{d(10)}{dx}$$

$$4\frac{d(x)}{dx} - 5\frac{d(y)}{dx} = \frac{d(10)}{dx}$$

$$4 - 5\frac{dy}{dx} = 0$$

Step 2: Solve algebraically for dy/dx

$$-5\frac{dy}{dx} = -4$$

$$\frac{dy}{dx} = \frac{-4}{-5} = \frac{4}{5}$$

Note: As we saw above in the case of 5y, when we have to differentiate y with respect to x, the derivative is dy/dx. Take another example, suppose we have to differentiate y^5. Then first differentiate it using the first rule and we get $5y^{5-1}$, that is $5y^4$. Now write the derivative of y. So the final answer is $5y^4\frac{dy}{dx}$.

Eg. Find the derivative of the following functions using rule for differentiation of an implicit function

$$5x^2 - y^3 = 10$$

Differentiating both sides $5\frac{dx^2}{dx} - \frac{dy^3}{dx} = \frac{d10}{dx}$

$$10x - 3y^2\frac{dy}{dx} = 0$$

Solving for $\frac{dy}{dx}$, $-3y^2\frac{dy}{dx} = -10x$

$$\frac{dy}{dx} = \frac{-10x}{-3y^2} = \frac{10x}{3y^2}$$

2. $5x^2 - 5y^3 = 20$

$$5\frac{dx^2}{dx} - 5\frac{dy^3}{dx} = \frac{d20}{dx}$$

$$5 \times 2x - 5 \times 3y^2\frac{dy}{dx} = 0$$

$$10x - 15y^2\frac{dy}{dx} = 0$$

$$-15y^2\frac{dy}{dx} = -10x$$

$$\frac{dy}{dx} = \frac{-10x}{-15y^2} = \frac{10x}{15y^2}$$

3. xy =15

Differentiating both sides, we get $\frac{d(xy)}{dx} = \frac{d(1.5)}{dx}$

We have to use Rule 5 (Product rule) to differentiate $\frac{d(xy)}{dx}$

$$x\frac{d(y)}{dx} + y\frac{d(x)}{dx} = \frac{d(1.5)}{dx}$$

$$x\frac{d(y)}{dx} + y = 0$$

$$x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

4. $x^2 + xy + y = 1$

$$\frac{d(x)^2}{dx} + \frac{d(xy)}{dx} + \frac{d(y)}{dx} = \frac{d(1)}{dx}$$

We have to use Rule 5 (Product rule) to differentiate the second term $\frac{d(xy)}{dx}$

$$2x + x\frac{dy}{dx} + y + \frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx}(x+1) = -(2x+y)$$

$$\frac{dy}{dx} = \frac{-2x+y}{x+1}$$

Higher Order Derivatives

Consider a function $y = 5x^4 + x^6$. We know that $y^{|} = \frac{dy}{dx} = 20x^3 + 6x^5$.

The $y^{|} or \frac{dy}{dx}$ is called the first order derivative. But since $y^{|}$ is also a function of x, it can again be differentiated with respect to x, if we wish.

This can be represented as $y^{||}$ or $\frac{d^2y}{dx^2}$ which is called the second order derivative. If the second order derivative is a differentiable function, we can differentiate it further to get still higher order derivatives like third order derivative, fourth order derivative etc.

$y^{||}$ or $\frac{d^2y}{dx^2} = \frac{d\frac{dy}{dx}}{dx} = \frac{d20x^3}{dx} + \frac{d6x^3}{dx} = 60x^2 + 30x^4$ → Second order derivative

Similarly

$y^{|||}$ or $\frac{d^3y}{dx^3} = 120x + 120x^3$→ Third order derivative

$y^{||||}$ or $\frac{d^4y}{dx^4} = 120 + 360$→ Fourth order derivative

This process may be continued up to the nth order which is represented as $y^n or \frac{d^ny}{dx^n}$

Examples: Find the second and third order derivatives of the following functions.

1. $y = 2x^3$

$$y^{|} = 6x^2,\ \ y^{||} = 12x,\ \ \ y^{|||} = 12$$

2. $y = 8x^5 + 7x$

$$y^{|} = 40x^4 + 7,\ \ y^{||} = 160x^3,\ \ y^{|||} = 480x^2$$

3. $y = 7x^4 + 5x^3 + 12x^2$

$$y^{|} = 28x^3 + 15x^2 + 24x$$

$$y^{||} = 84x^2 + 30x + 24$$

$$y^{|||} = 168x + 30$$

Maxima and Minima

To distinguish between maxima and minima also we can make use of second order derivatives.

Consider figure 6.

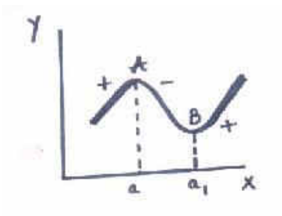


Fig 6

In figure 6 the function has a maximum value at x = a, since value of the function at x=a - that is point A – is greater than all a values in the immediate neighbourhood of the point . Loosely speaking the function is at maximum at x = a since the curve is rising at the left of A and it is falling to the right of A. Similarly, the function has a minimum value at x = a_1, since the value of function at x = a_1 -that is point B- is less than all values in the immediate neighbourhood of the point. Loosely speaking the function is at minimum at x = a_1, since the curve is falling to the left of B and it is rising to the right of B. The maximum and minimum values are together called the extreme values of the function.

We have already seen that at maximum or minimum the first derivative is equal to zero. If we take the second derivative, it can tell us what happens to the function after this point. Suppose the second derivative is –ve or less than zero (a –ve value of the derivatives represents a decreasing function). This means that the curve falls after the point. Naturally that point must be a maximum point. So for a point to be maximum, the first derivative should be equal to zero and the second derivatives should be less than zero. Similarly suppose that the second derivatives is +ve or greater than zero (a +ve value of the derivatives represents an increasing function). This means that the curves rise after the point. Naturally that point must be a minimum point. So for a point to be a minimum, the first derivative should be equal to zero and the second derivative should be less than zero or –ve. Often, the conditions to be satisfied for a maximum or minimum, by the first derivatives is called the first order condition and by the second derivative is called the second order condition. To summarize, for a condition y = f(x), the condition for a maxima or minima to occur at x = a are

	First order condition	Second order condition
Maxima	$y^{I}(a) = 0$	$y^{II}(a) < 0$
Minima	$y^{I}(a) = 0$	$y^{II}(a) < 0$ $y^{II}(a) >$

Note: if $y^{II} = 0$, it represents an inflexion point.This is out of the preview of the syllabus.

The concept of maxima and minima has wide application in economics. For instance it can help a consumer to determine the quantity to be purchased in order to maximize his utility and a producer to determine the quantity to be produced in order to maximize his profit or maxims his profit. Such exercises require practical methods finding maxima and minima, which are discussed in the following section.

Optimization of Functions

Optimization is the process of finding the relative maximum or minimum of a function. This can be done through the following steps.

Step 1: (First order condition or first derivative test). Take thefirst order derivative.

Set it equal to zero. Solve and find the critical values.

Step 2: (Second order condition or second order derivative test). Take the second derivative, evaluate it at the critical values and check the signs. If at a critical value a,

$$f^{II}(a) > 0: relative\ minimum$$

$$f^{II}(a) < 0: relative\ maximum$$

Example

E.g.1 Examine the following function for its maximum or minimum (or the question can be 'optimize the following function').

$y = 4x - x^2$

Step 1. Take the first order derivative.

$$\frac{dy}{dx} = y^{|} = 4 - 2x$$

Equating to zero

$4 - 2x = 0$

Solve and find the critical values that are find value of x.

$$-2x = -4$$

$$2x = 4$$

$$x = \frac{4}{2} = 2 \quad (critical\ value)$$

Now we have to see at x=2 whether the value of y is at a maximum or minimum. To understand this we have to check second order condition.

Step 2: Take the second order derivative

$$\frac{d^2y}{dx^2} = y^{||} = -2$$

Evaluate it at the critical values

$$y^{||}(x = 2) = -2$$

and check the signs

$$y^{||}(x = 2) = -2 < 0$$

Thus the function has a maximum value at x = 2.

This maximum value of y can be obtained by substituting the value of x in the given function

$y(2) = 4(2) – 22 = 8 – 4 = 4$.

Thus the function $y = 4x – x2$ is at a maximum when $x = 2$ and the maximum value of the function is 4.

E.g. 2: Optimize the function $y = x^2 – 4x + 8$

Step 1. $y^{|} = 2x – 4$. Equating to zero $2x – 4 = 0$.

Solving for x, $2x = 4$.

$x = 2$ (critical value).

Now is value of y a minimum or a maximum at $x = 2$. To understand this we have to check the second order condition.

Step 2. $y^{||} = 2$

$y^{||}(2) = 2 > 0$

Thus the function has a minimum value is $y(2) = 22- 4 (2) = 8 = 4$. Thus the function is at a minimum when $x = 2$ and the minimum value of the function is 4.

E.g. 5: Average cost of firms is given by $AC = 50 – 6q + q^2$. Where q = quantity find (a) the output level at which AC is minimum (b) the minimum AC at this output.

Step 1. $AC^{!} = -6 + 2q$

Equating to zero, $-6 + 2q = 0$, $q = 3$. $q = 3$ is the critical value . Now we have to see whether the function is at a minimum at $v = 3$. For this we have to check the sign of AC”(3).

Step 2. $AC^{||} = 2$.

$AC^{||}(3) = 2 > 0$ So the function (AC) is at a minimum when output is 3. In order to get the minimum AC we have to find AC(3).

(a) $AC(3) = 40 – 6(3) + 3^2 = 41$. So the AC is at a minimum when the output is 3 and the minimum AC is (Rs) 41.

Curvature Properties-Convexity and Concavity

A formal definition of concavity and convexity of a function is given by Edward. T. Doweling as follows. A function f(x) is concave at x = a if in some small region very close to this point the graph of the function lies completely below its tangent line. A function is convex at x = a if in the area very close to this point the graph of the function lies completely above its tangent line. We can distinguish between the concavity and the convexity using second order derivatives as follows.

(1). If, $f^{||}(a) > 0$, (that is +ve), the function is convex at the point $x = a$

(2). If, $f^{||}(a) < 0$, (that is –ve), the function is concave at the point

Note that the first order derivatives are irrelevant in the distinguishing between concavity and convexity.

E.g. Examine whether the following functions are concave or convex at x = 2

(1) $y = 2x^2 + 14x$
$y^{|} = 4x + 14$, $y^{||} = 4$.

Evaluate at x= 2. Since there is no x at the RHS, we need not do any substitution, simply write $Y^{||}(x=2)$.

$Y^{||}(x=2) = 4 > 0$ Hence the function is convex at x = 2.

(2) $y = -2x^2 + 14x$
$y^{|} = -4x + 14$, $y'' = -4$

Evaluate at x= 2. Since there is no x at the RHS, we need not do any substitution, simply write $Y^{||}(x=2)$.

$Y^{||}(x=2) = -4 < 0$ Hence the function is convex at x = 2.

Application of derivatives in economics – Marginal Concepts

Derivatives can be used to find the various marginal concepts like marginal cost (MC), marginal Revenue (MR) etc. from their total concepts like Total Cost, Total Revenue etc. We know that MC represents the change in total cost (TC) due to the production of an additional unit of the good. We also know that TC = f(q) where q is output. Then if we differentiate TC with respect to q, it will give us the change in TC due to a small change in output, that is MC. Thus

$$MC = \frac{dTC}{dq}$$

Thus MC is the first order derivative of the cost function with respect to output. Similarly Marginal Revenue (MR) represents the change in Total Revenue (TR) due to the production of an additional unit of the good. We also know that TR = f(q), where q is output. Then if we differentiate TR with respect to q, it will give us the change in TC due to a small change in output that is MR. Thus

Thus MR is the derivatives of the revenue function with respect to output or sales. Similarly we can find the marginal concept of any function by differentiating the respective total concept.

$$MR = \frac{dTR}{dq}$$

E.g. 1 If $TC = 20q + 5q^2$, find MC.

$$MC \; \frac{dTC}{dq} = \frac{d(20q + 5q2}{dq} = 20 + 10q$$

E.g. 2 If $TR = q^2 + 8q + 40$, find MR.

$$MR = \frac{dTR}{dq} = \frac{d(q2 + 8q + 40)}{dq} = 2q + 8$$

E.g. 3 Given a demand function q = 100 – 5p, find MR.

Here we have to form the TR function .We know that TR = p x q. We can derive p from the given demand function as p= 20 – 0.2 q. Assume that the quantity is 'q' .Then

TR = (20 – 0.2 q)q

TR = 20 q – 0.2 q2

$$MR = \frac{dTR}{dq} = \frac{d(20q - 0.2q2}{dq} = 20 - 0.4q$$

Application of derivatives in economics –Elasticity

Derivatives can be used to explain the different concept of elasticity like price, income and cross elasticities. However, our present knowledge of derivatives is adequate only for explaining price elasticity of demand. Other concepts will be explained after we develop practical derivatives.

<u>Price Elasticity of Demand</u>.

We know that the price elasticity of demand is a measure of the responsiveness of demand to change in the commodities own price. If the price changes are very small, we use the method of point elasticity to measure elasticity. The point elasticity of demand is expressed as the percentage change in the quantity demeaned divided by the percentage change in price.

Price elasticity of demand is a measure of the relative change in the amount demanded in response to change in price. It is the responsiveness of demand for a product following a change in its own price. The coefficient of price elasticity of demand is often represented by Greek alphabet η or simply e_p. The formula for calculating the co-efficient of elasticity of demand is: Percentage change in quantity demanded divided by the percentage change in price

$$\eta = e_p = \frac{\Delta Q_d / Q_d}{\Delta P / P} = \frac{\Delta Q_d}{\Delta P} \frac{P}{Q_d}$$

Since changes in price and quantity nearly always move in opposite directions, economists usually do not bother to put in the minus sign. We are concerned with the co-efficient of elasticity of demand. Because the calculation uses proportionate changes, the result is a unit less number and does not depend on the units in which the price and quantity are expressed. Also note that the price elasticity of demand is the most popular of all the elasticity concepts. So we often refer to it as simply elasticity of demand.

If η = 0 then demand is said to be perfectly inelastic. This means that demand does not change at all when the price changes – the demand curve will be vertical. For e.g. a 1% fall in price would lead to a no change in quantity demanded.

If η is between 0 and 1 (i.e. the percentage change in demand is smaller than the percentage change in price), then demand is inelastic. Here the change in demand will be proportionately smaller than the percentage change in price. For e.g. a 1% fall in price would lead to a less than 1% increase in quantity.

If η = 1 (i.e. the percentage change in demand is exactly the same as the percentage change in price), then demand is said to unit elastic. For e.g. a 1% fall in price would lead to exactly 1% rise in quantity.

If $\eta > 1$, then demand responds more than proportionately to a change in price i.e. demand is elastic. For e.g. a 1% fall in the price lead to more than 1 % increase in quantity.

If $\eta = \infty$, the demand is perfectly elastic. This implies a small change in price results in extremely huge change in quantity. For e.g. a 1% fall in the price lead to infinite (very large) increase in quantity.

Thus the value of price elasticity of demand stretches between 0 (perfectly inelastic demand)and ∞ (perfectly elastic demand).

In terms of calculus, $\eta = \frac{dQ}{dP}\frac{P}{Q}$

Example: **1.** Given the demand function q = −5p + 100, find price elasticity of demand when price is equal to 5.
We use the formula with calculus

$$\eta = \frac{dQ}{dP}\frac{P}{Q}$$

given $q = -5p + 100$, $\frac{dq}{dp} = -5$

when $P = 5$, $Q = -5(5) + 100 = 75$

substituting

$\eta = -5\ \frac{5}{75} = -0.33 = 0.33$ (since we discard the sign in measurement of price elasticity)

<u>Application of derivatives in economics</u> – Optimization (This topic we have already discussed under Maxima and Minima)

Module II

Index Numbers and Time Series Analysis

Index Numbers: Meaning and Uses- Unweighted and Weighted Index Numbers: Laspeyre‘s,

Paasche‘s, Fisher‘s, Dorbish-bBowley, Marshall-Edgeworth and Kelley‘s Methods - Tests of Index Numbers: Time Reversal and Factor Reversal tests. Base Shifting, Splicing and Deflating. Special Purpose Indices - Wholesale Price Index, Consumer Price Index and Stock Price Indices: BSESENSEX and NSE-NIFTY.

Time Series Analysis-Components of Time Series, Measurement of Trend by Moving Average and the Method of Least Squares.

Index Numbers

In simple terms, an index (or index number) is a number showing the level of a variable relative to its level (set equal to 100) in a given base period. It is a number that expresses the relative change in price, quantity, or value compared to a base period. Index numbers measure the value of an item (or group of items) at a particular point in time, as a percentage of the value of an item (or group of items) at another point in time.

In other words Index numbers are the indicators which measure percentage changes in a variable (or a group of variables) over a specified time. For example, if we say that the index of export for the year 2013 is 125, taking base year as 2010, it means that there is an increase of 25% in the country's export as compared to the corresponding figure for the year 2000.

Compiling index numbers is not a recent innovation. An Italian, G. R. Carli, is credited with originating index numbers in 1764. They were incorporated in a report he made regarding price fluctuations in Europe from 1500 to 1750. No systematic approach to collecting and reporting data in index form was evident until about 1900. The cost-of-living index (now called the Consumer Price Index) was introduced in 1913, and a long list of indexes hasbeen compiled since then.

Definitions of Index number

We now see some formal definitions of Index numbers.

Bowley: “Index numbers are used to measure the changes in some quantity which we cannot observe directly”

Patternson: “In its simplest form, an index number is the ratio of two index numbers expressed as a percent. An index is a statistical measure, a measure designed to show changes in one variable or a group of related variables over time, with respect to geographical location or other characteristics.”

Spiegel: “An index number is a statistical measure, designed to measure changes in a variable, or a group of related variables with respect to time, geographical location or other characteristics such as income, profession, etc.”

We can thus say that index numbers are economic barometers to judge the inflation (increase in prices) or deflationary (decrease in prices) tendencies of the economy. They help the government in adjusting its policies in case of inflationary situations.

TYPES OF INDEX NUMBERS

Index numbers are names after the activity they measure. Their types are as under:

Price Index: Measure changes in price over a specified period of time. It is basically the ratio of the price of a certain number of commodities at the present year as against base year.

Quantity Index: As the name suggest, these indices pertain to measuring changes in volumes of commodities like goods produced or goods consumed, etc.

Value Index: A value index measures changes in both the price and quantities involved. A value index, such as the index of department store sales, needs the original base-year prices, the original base-year quantities, the present-year prices, and the present-year quantities for its construction.

Purpose of Index Numbers

Indices of the elementary kind have little value in themselves. But they can be used to compile more complex "composite" indices, involving many different goods and services. In economic statistics, the term "index numbers" is usually reserved for these more complex "composite" indices. Index numbers become essential when measuring the change in price of a whole variety of products, the prices of which may be varying in different ways.

An index number, which is designed keeping, specific objective in mind, is a very powerful tool. For example, an index whose purpose is to measure consumer price index, should not include wholesale rates of items and the index number meant for slum-colonies should not consider luxury items like A.C., Cars refrigerators, etc.

Index numbers are meant to study the change in the effects of such factors which cannot be measured directly. For example, changes in business activity in a country are not capable of direct measurement but it is possible to study relative changes in business activity by studying the variations in the values of some such factors which affect business activity, and which are capable of direct measurement.

CHARACTERISTICS OF INDEX NUMBERS

Following are some of the important characteristics of index numbers:

(a) Index numbers are expressed in terms of percentages to show the extent of relative change

(b) It has a base period

(c) Index numbers measure relative changes. They measure the relative change in the value of a variable or a group of related variables over a period of time or between places.

(d) Index numbers measures changes which are not directly measurable.

(e) It facilitates the comparison of unlike series

(f) If the numbers are very large, often it is easier to comprehend the change of the index than the actual numbers.

The cost of living, the price level or the business activity in a country are not directly measurable but it is possible to study relative changes in these activities by measuring the changes in the values of variables/factors which affect these activities.

PROBLEMS IN THE CONSTRUCTION OF INDEX NUMBERS

The decision regarding the following problems/aspect has to be taken before starting the actual construction of any type of index numbers.

(i) Purpose of Index numbers under construction

(ii) Selection of items

(iii) Choice of an appropriate average

(iv) Assignment of weights (importance)

(v) Choice of base period

Let us discuss these one-by-ones

(i) Purpose of Index Numbers

An index number, which is designed keeping, specific objective in mind, is a very powerful tool. For example, an index whose purpose is to measure consumer price index, should not include wholesale rates of items and the index number meant for slum-colonies should not consider luxury items like A.C., Cars refrigerators, etc.

(ii) Selection of Items

After the objective of construction of index numbers is defined, only those items which are related to and are relevant with the purpose should be included.

(iii) Choice of Average

As index numbers are themselves specialized averages, it has to be decided first as to which average should be used for their construction. The arithmetic mean, being easy to use and calculate, is preferred over other averages (median, mode or geometric mean). In this lesson, we will be using only arithmetic mean for construction of index numbers.

(iv) Assignment of weights

Proper importance has to be given to the items used for construction of index numbers. It is universally agreed that wheat is the most important cereal as against other cereals, and hence should be given due importance.

(v) Choice of Base year

The index number for a particular future year is compared against a year in the near past, which is called base year. It may be kept in mind that the base year should be a normal year and economically stable year.

USES OF INDEX NUMBERS

Index Numbers have today become one of the most widely used measure for judging the pulse of economy, although in the beginning they were originally constructed to measure the effect of changes in prices. Today we use index numbers for cost of living, industrial production, agricultural production, imports and exports, etc

Index numbers are commonly used statistical device for measuring the combined fluctuations in a group related variables. If we wish to compare the price level of consumer items today with that prevalent ten years ago, we are not interested in comparing the prices of only one item, but in comparing some sort of average price levels. We may wish to compare the present agricultural production or industrial production with that at the time of independence. Here again, we have to consider all items of production and each item may have undergone a different fractional increase (or even a decrease). How do we obtain a composite measure? This composite measure is provided by index numbers which may be defined as a device for combining the variations that have come in group of related variables over a period of time, with a view to obtain a figure that represents the ‘net’ result of the change in the constitute variables.

Index numbers may be classified in terms of the variables that they are intended to measure. In business, different groups of variables in the measurement of which index number techniques are commonly used are (i) price, (ii) quantity, (iii) value and (iv) business activity. Thus, we have index of wholesale prices, index of consumer prices, index of industrial output, index of value of exports and index of business activity, etc. Here we shall be mainly interested in index numbers of prices showing changes with respect to time, although methods described can be applied to other cases. In general, the present level of prices is compared with the level of prices in the past. The present period is called the current period and some period in the past is called the base period.

1) Index numbers are used as economic barometers:

   Index number is a special type of averages which helps to measure the economic fluctuations on price level, money market, economic cycle like inflation, deflation etc. G.Simpson and F.Kafka say that index numbers are today one of the most widely used statistical devices. They are used to take the pulse of economy and they are used as indicators of inflation or deflation tendencies. So index numbers are called economic barometers.

2) Index numbers helps in formulating suitable economic policies and planning etc.

   Many of the economic and business policies are guided by index numbers. For example while deciding the increase of DA of the employees; the employer's have to depend primarily on the cost of living index. If salaries or wages are not increased according to the cost of living it leads to strikes, lock outs etc. The index numbers provide some guide lines that one can use in making decisions.

3) They are used in studying trends and tendencies.

   Since index numbers are most widely used for measuring changes over a period of time, the time series so formed enable us to study the general trend of the phenomenon under study. For example for last 8 to 10 years we can say that imports are showing upward tendency.

4) They are useful in forecasting future economic activity.

   Index numbers are used not only in studying the past and present workings of our economy but also important in forecasting future economic activity.

5) Index numbers measure the purchasing power of money.

   The cost of living index numbers determine whether the real wages are rising or falling or remain constant. The real wages can be obtained by dividing the money wages by the corresponding price index and multiplied by 100. Real wages helps us in determining the purchasing power of money.

6) Index numbers are used in deflating.

   Index numbers are highly useful in deflating i.e. they are used to adjust the wages for cost of living changes and thus transform nominal wages into real wages, nominal income to real income, nominal sales to real sales etc. through appropriate index numbers.

Limitations of Index numbers

However, the index numbers are not a faultless guide. They suffer from a number of limitations, some of which are given below:

1. They are approximations: They cannot be taken as infallible guides. Their data are open to question and they lead to different interpretations.

2. International comparisons are difficult, if not impossible, on account of the different bases, different sets of commodities or difference in their quality or quantity.
3. Comparison between different times are also not easy. Over long periods, some popular commodities are replaced by others. Entirely new commodities come to figure in consumption or a commodity may be vastly different from what it used to be.
4. Index numbers measure only changes in the sectional price levels. An index number that helps us to study the economic conditions of mill hands or railway coolies will be useless for a study of the conditions of college lecturers. An entirely different set of commodities will have to be selected. Different people use different things and hold different assets. Therefore, different classes of people are affected differently by a given change in the price level. Hence, the same index number cannot throw light on the effects of price changes on all sections of society.

Methods of Constructing Index Numbers

Construction of index numbers can be divided into two types:

(a) <u>Unweighted indices</u>

(i) Simple Aggregative method
(ii) Simple average of price relative method

(b) <u>Weighted indices</u>

(i) Weighted Aggregative Indices

1. Laspayers Method
2. Paashe Method
3. Dorbish&Bowley's method
4. Fisher's ideal Method
5. Marshall –Edgeworth Method, and
6. Kelley's Method

(ii) Weighted Average of relatives

Let us see them in detail.

a (i) Simple Aggregative Method
This is a simple method for constructing index numbers. In this method, the total of the prices of commodities in a given (current) years is divided by the total of the prices of commodities in a base year and expressed as percentage.

$$P_{01} = \frac{\sum P_0}{\sum P_1} \times 100$$

$\sum P_1$ = Total of Current year prices for various commodities

$\sum P_0$ = Total of base year prices for various commodities

Example 1

Let us take an example to illustrate

Construct the price index number for 2013, taking the year 2010 as base year

Commodity	Price in the year 2010	Price in the year 2013
A	60	80
B	50	60
C	70	100
D	120	160
E	100	150

Solution :

Calculation of simple Aggregative index number for 2013 (against the year 2010) using the formula.

Commodity	Price in the year 2010 P_0	Price in the year 2013 P_1
A	60	80
B	50	60
C	70	100
D	120	160
E	100	150
	$\sum P_0 = 400$	$\sum P_1 = 550$

Substitute in the formula

$$P_{01} = \frac{\sum P_0}{\sum P_1} \times 100 = \frac{400}{550} \times 100 = 137.50$$

This means that the price index for the year 2013, taking 2010 as base year, is 137.5, showing that there is an increase of 37.5% in the prices in 2013 as against 2010.

Example 2

Compute the index number for the years 2011, 2012, 2013 and 2014, taking 2010 as base year, from the following data.

Year	2010	2011	2012	2013	2014
Price	120	144	168	204	216

Solution :
Price relatives for different years are

Year	2010	2011	2012	2013	2014

Price	$\frac{120}{120} \times 100$ $= 100$	$\frac{144}{120} \times 100$ $= 120$	$\frac{168}{120} \times 100$ $= 140$	$\frac{204}{120} \times 100$ $= 170$	$\frac{216}{120} \times 100$ $= 180$

Price indexes for different years are as in the following table.

Year	2010	2011	2012	2013	2014
Price Index	100	120	140	170	180

There are two main limitations of this method. They are;

(i)The units used in the price or quantity quotations can exert a big influence on the value of the index, and

(ii) No consideration is given to the relative importance of the Commodities.

a (ii) Simple Average of price Relatives Method

Price Relative means the ratio of price of a certain item in current year to the price of that item in base year, expressed as a percentage (i.e. Price Relative = $(p_2/p_1)\times100$). For example, if a fridge TV cost Rs 12000 in 2005 and Rs. 18000 in 2013, the price relative is (18000/12000)×100 = 150.

When this method is used to construct a price index, first of all price relatives are obtained for the various items included in the index and then arrange of these relatives is obtained using any one of the measures of central value, ie, arithmetic mean, median, mode, geometric or harmonic mean. When arithmetic mean is used for averaging the relatives, the formula for computing the index is:

$$P_{01=}\frac{\Sigma\left(\frac{P_1}{P_0}\times 100\right)}{N}$$

if A.M. is used as average where P_{01} Is the price index, N is the number of items,P_0 is the price in the base year and P_1 is the price of corresponding commodity in present year (for which index is to be calculated).

Example: Construct by simple average of price relative method the price index of 2013, taking 2010 as base year from the following data

Commodity	A	B	C	D	E	F
Price in 2010	60	50	60	50	25	20
Price in 2014	80	60	72	75	37.5	30

Solution

Find the price relatives for each take the sum, substitute in formula.

Commodity	A	B	C	D	E	F
Price in 2010 (P_0)	60	50	60	50	25	20
Price in 2014 (P_1)	80	60	72	75	37.5	30
Price relative $\frac{P_1}{P_0} \times 100$	$\frac{60}{80} \times 100$	$\frac{60}{50} \times 100$	$\frac{72}{60} \times 100$	$\frac{75}{50} \times 100$	$\frac{37.5}{25} \times 100$	$\frac{30}{20} \times 100$
	133.33	120.00	120.00	150.00	150.00	150.00
	$\sum \frac{P_1}{P_0} \times 100 = 823.33$					

Substituting we get

$$P_{01=} \frac{\Sigma\left(\frac{P_1}{P_0} \times 100\right)}{N} = \frac{823.33}{6} = 137.22$$

Price index for 2013, taking 2010 for base year = 137.22

An un-weighted aggregate price index represents the changes in prices, over time, for an entire group of commodities. However, an un-weighted aggregate price index has two short comings. First, this index considers each commodity in the group as equally important. Thus, the most expensive commodities per unit are overly influential. Second, not all the commodities are consumed at the same rate. In an un-weighted index, changes in the price of the least consumed commodities are overly influential.

(b) i. Weighted Aggregative Indices

Due to the shortcomings of un-weighted aggregate price indices, weighted aggregate price indices are generally preferable. Weighted aggregate price indices account for differences in the magnitude of prices per unit and differences in the consumption levels of the items in the market basket.

When all commodities are not of equal importance, this method is used. Here we assign weight to each commodity relative to its importance and index number computed from these weights is called weighted index numbers.

b.i. (i) 1. Laspayers Method

In this index number the base year quantities are used as weights, so it also called base year weighted index.

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0\, q_0} \times 100$$

The primary disadvantage of the Laspeyres Method is that it does not take into consideration the consumption pattern. The Laspeyres Index has an upward bias. When the prices increase, there is a tendency to reduce the consumption of higher priced items. Similarly when prices decline, consumers shift their purchase to those items which decline the most.

b. i. (ii) Paasche's Method

Under this method weights are determined by quantities in the given year

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0\, q_1} \times 100$$

The Paasche price index uses the consumption quantities in the year of interest instead of using the initial quantities. Thus, the Paasche index is a more accurate reflection of total consumption costs at that point in time. However, there are two major drawbacks of the Paasche index. First, accurate consumption values for current purchases are often difficult to obtain. Thus, many important indices, such as the consumer price index (CPI), use the Laspeyres method. Second, if a particular product increases greatly in price compared to the other items in the market basket, consumers will avoid the high-priced item out of necessity, not because of changes in what they might prefer to purchase.

b.i. (iii) Dorbish&Bowley's Method

Dorbish and Bowley have suggested simple arithmetic mean of the two indices (Laspeyres and Paasche) mentioned above so s to take into account the influence of both the periods, i.e., current as well as base periods. The formula for constructing the index is:

$$P_{01} = \frac{L + P}{2}$$

Where L = Laspeyres Index P = Paasche's Index

OR it may be written as

$$P_{01} = \frac{\frac{\sum p_1 q_0}{\sum p_0\, q_0} + \frac{\sum p_1 q_1}{\sum p_0\, q_1}}{2} \times 100$$

b.i. (iv) Fisher's Ideal Index

The geometric mean of Laspeyre's and Paasche's price indices is called Fisher's price Index. Fisher price index uses both current year and base year quantities as weight. This index corrects the positive bias inherent in the Laspeyres index and the negative bias inherent in the Paasche index. Fisher's price index is also a weighted aggregative price index because it is an average (G.M) of two weighted aggregative indices. The computational formula for the fisher ideal price index is:

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0\, q_0} \times \frac{\sum p_1 q_1}{\sum p_0\, q_1}} \times 100$$

OR

$$P_{01} = \sqrt{L \times P}$$

Fischer’s Index is known as ‘ideal’ because (1) it is based on geometric mean, which is considered to be the best average for constructing index numbers. (2) It takes into account both current as well as base year prices and quantities (3) It satisfies both time reversal as well as the factor reversal tests (which we will study soon) and (4) it is free from bias.

It is not, however, a practical index to compute because it is excessively laborious. The data, particularly for the Paasche segment of the index, are not readily available.

b.i. (v) Marshall-Edgeworth Method

If the weights are taken as the arithmetic mean of base and current year quantities, then the weighted aggregative index is called Marshal-Edgeworth index. Like Fisher’s index, Marshall-Edgeworth index also requires too much labour in selection of commodities. In some cases the usage of this index is not suitable, for example the comparison of the price level of a large country to a small country. Marshal-Edgeworth index can be calculated by using the formula given below.

$$P_{01} = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

It is a simple, readily constructed measure, giving a very close approximation to the results obtained by the ideal formula.

The Marshall-Edgeworth formula uses the arithmetic mean of the quantities purchased in the base and current periods as weights. Like the Fisher 'Ideal' index it is impracticable to use as a timely indicator of price change because it requires the use of quantities purchased in the current period. In practice, the Marshall-Edgeworth index and the Fisher Ideal, index give similar results.

b.i. (vi) Kelley’s Method

According to Truman L. Kelly the formula for constructing index numbers is:

$$P_{01} = \frac{\sum p_1 q}{\sum p_0 q} \times 100$$

Where q refer to some period, not necessarily the base year or current year.

Example 1: From the following data calculate Price Index Numbers for 2000 with 2013 as base year by using: (i) Laspayers Method (ii) Paasche’s Method (iii) Dorbish&Bowley’s Method (iv) Fisher’s Ideal Index (v) Marshall-Edgeworth Method

	2000		**2013**	
Commodity	**Price**	**Quantity**	**Price**	**Quantity**
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

Solution

Let us first compute the necessary values.

(i) Laspayers Method

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0\, q_0} \times 100$$

	2000		2013			
Commodity	**P_0**	**Q_0**	**P_1**	**Q_1**	**P_1Q_0**	**P_0Q_0**
A	20	8	40	6	320	160
B	50	10	60	5	600	500
C	40	15	50	15	750	600
D	20	20	20	25	400	400
					2070	1660

$$P_{01} = \frac{2070}{1660} \times 100 = 124.70$$

(ii) Paasche's Method

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0\, q_1} \times 100$$

	2000		2013			
Commodity	**P_0**	**Q_0**	**P_1**	**Q_1**	**P_1Q_1**	**P_0Q_1**
A	20	8	40	6	240	120
B	50	10	60	5	300	250
C	40	15	50	15	750	600
D	20	20	20	25	500	500
					1790	1470

$$P_{01} = \frac{1790}{1470} \times 100 = 121.77$$

(iii) Dorbish&Bowley's Method

$$P_{01} = \frac{L+P}{2}$$

$$P_{01} = \frac{124.70 + 121.77}{2} = \frac{246.47}{2} = 123.23$$

(iv)Fisher's Ideal Index

$$P_{01} = \sqrt{L \times P}$$

$$P_{01} = \sqrt{124.70 \times 121.77} = \sqrt{15184.56} = 123.23$$

(v)Marshall-Edgeworth Method

$$P_{01} = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

	2000		**2013**					
Commodity	**P_0**	**Q_0**	**P_1**	**Q_1**	**P_1Q_1**	**P_0Q_1**	**P_1Q_0**	**P_0Q_0**
A	20	8	40	6	240	120	320	160
B	50	10	60	5	300	250	600	500
C	40	15	50	15	750	600	750	600
D	20	20	20	25	500	500	400	400
					1790	1470	2070	1660

$$P_{01} = \left(\frac{2070+1790}{1660+1470}\right) \times 100 = \frac{3860}{3130} \times 100 = 1.233226837 \times 100$$
$$= 123.32$$

Example 2: Compute index number from the following data

Materials	**Unit**	**Quantity required**	**Price**	
			2000	**2010**
Cement	100 *lb*	500 *lb*	5.0	8.0
Timber	*c.ft.*	2000 *c.ft.*	9.5	14.2
Steel	*Cwt.*	50 *cvt.*	34.0	42.20
Bricks	*Per 000*	20000	12.0	24.0

Solution

Since the quantities (weights) required of different materials are fixed for both base year and current year, we will use Kelly's formula.

For materials we have to do certain conversions. For example, for cement unit is in 100 lbs, and the quantity required is 500 lbs. Hence, the quantity consumed per unit for cement is 500/100 = 5. Similarly, the quantity consumed per unit for brick is 20000/1000= 20.

By Kelley's Method, $P_{01} = \frac{\sum p_1 q}{\sum p_0 q} \times 100$

Let us make the necessary computations.

Materials	Unit	Quantity required	q	Price (Rs.)		P_1q	P_0q
				2000 P0	2010 P1		
Cement	100 lb	500 lb	5	5.0	8.0	25	40
Timber	c.ft.	2000 c.ft.	2000	9.5	14.2	19000	28400
Steel	Cwt.	50 cvt.	50	34.0	42.0	1700	2100
Bricks	Per '000	20000	20	12.0	24.0	240	480
				Total		20965	31020

Substituting

$$P_{01} = \frac{\sum p_1 q}{\sum p_0 q} \times 100 = \frac{31020}{20965} \times 100 = \frac{1.4796}{100} = 147.96$$

B. (II) WEIGHTED AVERAGE OF RELATIVES

I. Weighted Average of Price Relatives Method

In this method, appropriate weights are assigned to the commodities according to the relative importance of those commodities in the group. Thus the index for the whole group is obtained on taking the weighted average of the price relatives. To find the average, Arithmetic Mean or Geometric Mean can be used.

When AM is used, the index is $\boldsymbol{P_{01}} = \frac{\sum PV}{\sum V}$

Where P = Price relative
V = Value of weights i.e. p_0q_0

Example:
From the following data compute price index by supplying weighted average of price relatives method using Arithmetic Mean

Commodity	Sugar	Flour	Milk
p_0	3.0	1.5	1.0
q_0	20 Kg.	40 Kg.	10 Lit.
p_1	4.0	1.6	1.5

By using Arithmetic Mean

Commodity	p_0	q_0	p_1	p_0q_0 (V)	$\frac{p_1}{p_0}$ x 100	PV
Sugar	3.0	20 Kg	4	60	$\frac{4}{3} \times 100$	8000
Flour	1.5	40 Kg.	1.6	60	$\frac{1.6}{1.5} \times 100$	6400
Milk	1.0	10 Lit.	1.5	10	$\frac{1.5}{1.0} \times 100$	1500
				$\sum V = 130$		$\sum PV = 15900$

$$P_{01} = \frac{\sum PV}{\sum V} = \frac{15900}{130} = 122.31$$

Instead of Arithmetic Mean, we can use Geometric Mean.

When GM is used, the index is $\boldsymbol{P_{01}} = Antilog\left\{\frac{\sum V\,logP}{\sum V}\right\}$

Where P $= \frac{P_1}{P_0}$ x 100

V = Value of weight

The above example can be re worked using GM as follows.

By using Geometric Mean

Commodity	p_0	q_0	p_1	p_0q_0 (v)	*p*	*Log p*	*V Log p*
Sugar	3.0	20 Kg	4	60	133.3	2.1249	127.494
Flour	1.5	40 Kg.	1.6	60	106.7	2.0282	121.692
Milk	1.0	10 Lit.	1.5	10	150.0	2.1761	21.761
				$\sum v = 130$			$\sum v.\,logp = 270.947$

$$P_{01} = Antilog\left\{\frac{\sum V\, logP}{\sum V}\right\} = Antilog\left[\frac{270.947}{130}\right] = Antilog\, 2.084 = 120.9$$

Merits of weighted Average of Relative Indices

- When different index numbers are constructed by the average of relatives method, all of which have the same base, they can be combined to form a new index.
- When an index is computed by selecting one item from each of the many sub groups of items, the values of each sub subgroup may be used as weights. Then only the method of weighted average of relatives is appropriate.
- When a new commodity is introduced to replace the one formerly used, the relative for the new it may be spliced to the relative for the old one, using the former value weights.
- The price or quantity relatives each single item in the aggregate are in effect, themselves a simple index that often yields valuable information for analysis.

TESTS OF INDEX NUMBERS

The following are the most important tests through which one can list the consistency of index numbers.

1. The time Reversal Test
2. The factor Reversal Test

1.The Time Reversal Test

$$P_{01} \times P_{10} = 1$$

Where P_{01} is the price index for year '1' with year '0' as base year and P_{10} is the price index for year 'a' with year 'b' as base.

This test is not satisfied by both Laspeyres and Paasche's index numbers.

$$P_{01} \times P_{10} = \frac{\sum p_1q_0}{\sum p_0q_0} \text{X} \frac{\sum p_0q_1}{\sum p_1q_1} \neq 1$$

Paasche's Method = $P_{01} \times P_{10} = \frac{\sum p_1q_1}{\sum p_0q_1} \text{X} \frac{\sum p_0q_0}{\sum p_1q_0} \neq 1$

Fisher's formula satisfies this test

Fisher's Method $= P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1} \times \frac{\sum p_0q_1}{\sum p_1q_1} \times \frac{\sum p_0q_0}{\sum p_1q_0}} = 1$

2. The Factor Reversal Test

$$P_{01} \times Q_{01} = \frac{\sum p_1q_1}{\sum p_0q_0}$$

Where P_{01} stands for the price relative for the year '1' with base year '0' and Q_{01} stands for quantity relative for the year '1' with base year '0', then the condition is

This test is not satisfied by both Laspeyres and Paasche's index numbers.

Laspeyre's Formula: $$P_{01} \times Q_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum q_1 p_0}{\sum q_0 p_0} \neq \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

Paasche's Formula: $$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum q_1 p_1}{\sum q_0 p_1} \neq \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

Fisher's formula satisfies this test

Fisher's Formula: $$P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}}$$

$$= \sqrt{\frac{\sum q_1 p_1{}^2}{\sum p_0 q_0}} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

Fisher's formula satisfies both time reversal and factor reversal test. This is why the Fisher's formula is often called Fisher's Ideal Index Number.

Example

For the following data prove that the Fisher's Ideal Index satisfies both the Time Reversal Test and the Factor Reversal Test.

Commodity	**Base Year**		**Current Year**	
	Price	**Quantity**	**Price**	**Quantity**
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24

Solution

	p_0	q_0	P_1	q_1	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
A	6	50	10	56	300	336	500	560
B	2	100	2	120	200	240	200	240
C	4	60	6	60	240	240	360	360
D	10	30	12	24	300	240	360	288
					$\sum p_0 q_0 = 1040$	$\sum p_0 q_1 = 1056$	$\sum p_1 q_0 = 1420$	$\sum p_1 q_1 = 1448$

Fisher's price index number is gven by

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

Substituting the values we get

$$P_{01} = \sqrt{\frac{1420}{1040} \times \frac{1448}{1056}} \times 100 = 136.83$$

Time reversal test: $\boldsymbol{P_{01} \times P_{10} = 1}$

We have P_{01} = 1.3683 (without factor 100)

And

$$P_{10} = \sqrt{\frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}} \ (without\ factor\ 100)$$

Substituting

$$P_{01} = \sqrt{\frac{1056}{1448} \times \frac{1040}{1420}} = 0.7308$$

$$p_{01} \times p_{10} = 1.3683 \times 0.7308 = 0.9999 \approx 1$$

Hence, Fischer's index satisfies Time Reversal Test.

Factor Reversal Test $\boldsymbol{P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}}$

We have (without factor 100)

$$Q_{01} = \sqrt{\frac{\sum p_0 q_1}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_1 q_0}} = \sqrt{\frac{1056}{1040} \times \frac{1448}{1420}}$$

$$P_{01} \times Q_{01} = \sqrt{\left(\frac{1420 \times 1448}{1040 \times 1056}\right) \times \left(\frac{1056 \times 1448}{1040 \times 1420}\right)}$$

$$= \sqrt{\left(\frac{1448}{1040}\right)^2} = \frac{1448}{1040} = \frac{\sum P_1 q_1}{\sum P_0 q_0}$$

Hence Fisher's Index satisfies Factor Reversal Test also.

BASE SHIFTING, SPLICING AND DEFLATING THE INDEX NUMBERS:

(a) Base shifting

Most index numbers are subjected to revision from time to time due to different reasons. In most cases it becomes compulsory to change the base year because numerous changes took place with the passage of time. For example changes may happen due to disappearance of old items, inclusion of new ones, changes in weights of commodities or changes in conditions, habits, and standard of life etc.

One of the most frequent operations necessary in the use of index numbers is changing the base of an index from one period to another without recompiling the entire series. Such a change is referred to as 'base shifting'. The reasons for shifting the base are

If the previous base has become too old and is almost useless for purposes of comparison.

If the comparison is to be made with another series of index numbers having different base.

The following formula must be used in this method of base shifting is

$$\text{Index number based on new base year} = \frac{\text{current years old index number}}{\text{new base years old index number}} \times 100$$

Shifting from one fixed base to another fixed base

To convert a fixed base to a new fixed base each old index is divided by the index of new base sought multiplied by 100. It can be illustrated with the help of following problem.

Example:

Following series is given to the base year 2000. Now convert it into the new series with base year 2003.

Year	2000	2001	2002	2003	2004	2005
Index	100	130	145	155	205	255

Year	**Fixed Base Index**	
	Base = 2000	**Base = 2003**
2000	100	100/155×100 = 64.52
2001	130	130/155×100 = 83.87
2002	145	145/155×100 = 93.55
2003	155	155/155×100 = 100.00
2004	205	205/155×100 = 132.26
2005	255	255/155×100 = 164.52

Shifting from chain base to fixed base

One of the disadvantages of chain base method is that the comparison between distant periods is not immediately evident. Therefore it becomes necessary to convert chain base indices into fixed base indices. This can be illustrated with the help of following example.

Example:

Convert the following chain indexes into the new series with base year 2005.

Year	2005	2006	2007	2008	2009	2010
Index	100	105	110	107	112	107

Year	**Chain Base Index**	**Fixed Index (1970 = 100)**
2005	100	100
2006	105	100×105/100 = 105
2007	110	105×110/100 = 115.5
2008	107	115.5×107/100 = 123.59
2009	112	123.59×112/100 = 138.42
2010	107	138.42×107/100 = 148.10

<u>Shifting from Fixed to chain base</u>

As discussed earlier, conditions change over a period due to revised weightings system, inclusion of new items and disappearance of old ones etc. Due to all these factors, sometimes it is necessary to convert the indices from fixed base to chain base. This can be explained with the help of following problem. Problem: Convert the following indexes with base 1980 to chain indexes.

Year	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
Fixed Index	100	105	115	130	150	175

Year		**Fixed Index (Base = 1980)**	**Chain Base Index** $P_{0,n} = P_N/P_0 \times 100$
2005	1980	100	$P_{2005,2005} = \frac{100}{100} \times 100 = 100$
2006	1981	105	$P_{2005,2006} = \frac{105}{100} \times 100 = 105$
2007	1982	115	$P_{2006,2007} = \frac{115}{105} \times 100 = 109.52$
2008	1983	130	$P_{2007,2008} = \frac{130}{115} \times 100 = 113.04$
2009	1984	150	$P_{2008,2009} = \frac{150}{130} \times 100 = 115.38$
2010	1985	175	$P_{2009,2010} = \frac{175}{150} \times 100 = 116.67$

Splicing of two series of index numbers

Splicing of index numbers mean combining two or more series of overlapping index numbers to obtain a single index number on a common base. This is done by the same technique as used in base shifting.

To combine two or more series of overlapping index numbers to obtain a single series of index numbers on a common base.
It is of two types:-
(i) Splicing of new index numbers to old index numbers
(ii) Splicing of old index numbers to new index number.
Splicing of Index numbers can be done only if the index numbers are constructed with the same items, and have an overlapping year. Suppose we have an index number with a base year of 2001 and another index number (using the same item as the first one) with a base of 2011. Suppose both index numbers are continuing. Then we can splice the first series of index number to the second series and have a common index with base 2011. We can also spice index number series two with series one and have a common index number with base 2001. Splicing is generally done when an old index number with an old base is being discontinued and a new index with a new base is being started.
The following formula must be used in this method of splicing
Index number after splicing =

$$\frac{\text{index number to be spliced} \times \text{old index number of existing base}}{100}$$

Example

Index Number A given below was started in 1981 and discontinued in 2001 when another index B was started which continues up to date. From the data given in the table below splice the index number B to index number A so that a continuous series of index numbers from 1951 up to date is available.

Splicing of Index B to Index A

Here we multiply index B with a common factor $\frac{200}{100}$ which is the ratio of index B to index A in the overlapping year 2001.

Year	**Index A**	**Index B**	**Index B Spliced to A**
1981 …. …. ….	100	-	-
2000	180	-	-
2001	200	100	$\frac{200}{100} \times 100 = 200$
2002	-	120	$\frac{200}{100} \times 120 = 240$
2003 … ….	-	140	$\frac{200}{100} \times 140 = 280$

….			
2013	-	250	$\frac{200}{100} \times 250 = 500$

Thus we have a continuous series of index numbers with base 1981 which continues up todate.

DEFLATING THE INDEX NUMBERS

By deflating we mean making allowances for the effect of changing price levels. A rise in price level means a reduction in the purchasing power of money. To take the case of a single commodity suppose the price of wheat rises from ₹ 500 per quintal in 1999 to ₹ 1,000 per quintal in 2009 it means that in 2009 one can buy only half of wheat if the spends the same amount which he was spending on wheat in 1999. Thus the value (or purchasing power) of a rupee is simply the reciprocal of an appropriate price index written as a proportion. If prices increase by 60 per cent, the price index is 1.60 and what a rupee will busy is only 1/1.60 or 5/8 of what it used to buy. In other words the purchasing power of rupee is 5/8 of what it was. Similarly, if prices increase by 25 per cent the price index is 1.25 (125 per cent). And the purchasing power of the rupee is 1/1.25 = 0.80.

Thus the purchasing power of money = $\frac{1}{price\ index}$

In times of rising prices the money wages should be deflated by the price index to get the figure of real wages. The real wages alone tells whether a wage earner is in better position or in worst position.

For calculating real wage, the money wages or income is divided by the corresponding price index and multiplied by 100.

i.e. Real wages = $\frac{Money\ wages}{\Pr ice\, index} \times 100$

Thus Real Wage Index= $\frac{\mathrm{Re}\, al\ wage\, of\ current\ year}{\mathrm{Re}\, al\ wage\, of\ base\ year} \times 100$

Example

The annual wage of workers (in Rs.) is given along with Consumer Price Indices. Find (i) the real wage and (ii) the real wage indices.

Year	2010	2011	2012	2013
Wages	1800	2200	3400	3600
Consumer Price Indices	100	170	300	320

Year	**Wage**	**Price Index**	**Real Wage**	**Real Wage Indices 2010 = 100**
2010	1800	100	$\frac{1800}{100} \times 100$ =1800	100
2011	2200	170	$\frac{2200}{170} \times 100$ =1294.1	$\frac{1294.1}{1800} \times 100$ =71.90

2012	3400	300	$\frac{3400}{300} \times 100$ =1133.3	$\frac{1133.3}{1800} \times 100$ =62.96
2013	3600	320	$\frac{3600}{320} \times 100$ =1125	$\frac{1125}{1800} \times 100$ =62.50

SPECIAL PURPOSE INDICES

Price Index: The price index is an indicator of the average price movement over time of a fixed basket of goods and services. The constitution of the basket of goods and services is done keeping in to consideration whether the changes are to be measured in retail, wholesale, or producer prices etc. The basket will also vary for economy-wide, regional, or sector specific series. At present, separate series of index numbers are compiled to capture the price movements at retail and wholesale level in India. There are four main series of price indices compiled at the national level. Out of these four, Consumer Price Index for Industrial Workers (CPI-IW), Consumer Price Index for Agricultural Labourers / Rural Labourers (CPI -AL/RL), Consumer Price Index for Urban Non-Manual Employees (CPI-UNME) are consumer price indices. The Wholesale Price Index (WPI) number is a weekly measure of wholesale price movement for the economy. Some states also compile variants of CPI and WPI indices at the state level.

1. Wholesale Price Index

The wholesale price index numbers indicate the general condition of the national economy. They measure the change in prices of products produced by different sectors of an economy. The wholesale prices of major items manufactured or produced are included in the construction of these index numbers.

Wholesale Price Index (WPI) represents the price of goods at a wholesale stage i.e. goods that are sold in bulk and traded between organizations instead of consumers. WPI is used as a measure of inflation in some economies.

Uses

In a dynamic world, prices do not remain constant. Inflation rate calculated on the basis of the movement of the Wholesale Price Index (WPI) is an important measure to monitor the dynamic movement of prices. As WPI captures price movements in a most comprehensive way, it is widely used by Government, banks, industry and business circles. Important monetary and fiscal policy changes are often linked to WPI movements. Similarly, the movement of WPI serves as an important determinant, in formulation of trade, fiscal and other economic policies by the Government of India. The WPI indices are also used for the purpose of escalation clauses in the supply of raw materials, machinery and construction work.

WPI is used as an important measure of inflation in India. Fiscal and monetary policy changes are greatly influenced by changes in WPI.

WPI is an easy and convenient method to calculate inflation. Inflation rate is the difference between WPI calculated at the beginning and the end of a year. The percentage increase in WPI over a year gives the rate of inflation for that year.

WPI computation in India

WPI is the most widely used inflation indicator in India. This is published by the Office of Economic Adviser, Ministry of Commerce and Industry. WPI captures price movements in a most comprehensive way. It is widely used by Government, banks, industry and business circles. Important monetary and fiscal policy changes are linked to WPI movements. It is in use since 1939 and is being published since 1947 regularly. We are well aware that with the changing times, the economies too undergo structural changes. Thus, there is a need for revisiting such indices from time to time and new set of articles / commodities are required to be included based on current economic scenarios. Thus, since 1939, the base year of WPI has been revised on number of occasions. The current series of Wholesale Price Index has 2004-05 as the base year.

Wholesale price index comprises as far as possible all transactions at first point of bulk sale in the domestic market. Provisional monthly WPI for All Commodities is released on 14th of every month (next working day, if 14th is holiday). Detailed item level WPI is put on official website (http://www.eaindustry.nic.in/) for public use. The provisional index is made final after a period of eight weeks/ two months.

The Office of the Economic Adviser to the Government of India undertook to publish for the first time, an index number of wholesale prices, with base week ended August 19, 1939 = 100, from the week commencing January 10, 1942. The index was calculated as the geometric mean of the price relatives of 23 commodities classified into four groups: (1) food & tobacco; (2) agricultural commodities; (3) raw materials; and (4) manufactured articles. Each item was assigned equal weight and for each item, there was a single price quotation. That was a modest beginning to what became an important weekly activity for the monitoring and management of the Indian economy and a benchmark for business transactions.

Step-in compilation of WPI in India

Like most of the price indices, WPI is based on Laspeyres formula for reason of practical convenience. Therefore, once the concept of wholesale price is defined and the base year is finalized, the exercise of index compilation involves finalization of item basket, allocation of weights (W) at item, groups/ sub-groups level. Simultaneously, the exercise to collect base prices (Po), current prices (P1), finalization of item specifications, price data sources, and data collection machinery is undertaken. These steps are

1. Definition of the Concept of Wholesale Prices:

Wholesale price has divergent connotations adopted by the different departments using them. There is no uniform definition for agricultural and non- agricultural commodities as all the wholesale prices cannot be collected from the established markets. So proper definition has to be given by the competent authority.

For example in the case of agricultural commodities, in practice, there are three types of wholesale markets viz., primary, secondary and terminal in the agricultural sector. The price movements and price levels in all three vary. Price movement in the terminal market may tend to converge toward the retail prices. Option to collect the wholesale prices for these three different stages of wholesale transactions exists for agricultural commodities though the primary market is prepared. So, the Ministry of Agriculture has defined wholesale price as the rate at which relatively large transaction of purchase, usually for further sale, is affected.

Similarly, for non-agricultural commodities, which are predominantly manufacturing items, the problem arises, as there are no established sources in markets. This is true of mining and fuel items also. The issue of ex-factory vis-à-vis wholesale prices for non-agriculture items have been discussed by the successive Working Groups set up for the revision of WPI and all have reached the conclusion that in practice, it is not feasible to collect wholesale prices for most of the manufacturing items. It has also been observed that the margin of wholesalers in case of non-agricultural commodities remains unchanged for over a long period of time. As a result, it is felt that the trends in the index compiled on the basis of ex-factory prices would not be much different from the index if compiled on the basis of wholesale prices if it were feasible to get these prices. The last Working Group has recommended collecting wholesale prices from the markets as far as possible, because the economy is moving towards globalization and open trade with inputs increasing in the commodities set.

2) Choice of Base Year

The second step is choice of base year. The well-known criteria for the selection of base year are (i) a normal year i.e. a year in which there are no abnormalities in the level of production, trade and in the price level and price variations, (ii) a year for which reliable production, price and other required data are available and (iii) a year as recent possible and comparable with other data series at national and state level. The National Statistical Commission has recommended that base year should be revised every five year and not later than ten years.

3. Selection of Items, Varieties/ Grades, Markets:

To ensure that the items in the index basket are as best representatives as possible, efforts are made to include all the important items transacted in the economy during the base year. The importance of an item in the free market will depend on its traded value during the base year. At wholesale level, bulk transactions of goods and services need to be captured. As the services are not covered so far, the WPI basket mainly consists of items from goods sector. In the absence of single source of data on traded value, the selection procedures followed for agricultural commodities and non-agricultural commodities have also been different.

For example, in the case of agricultural commodities: As there is a little scope of emergence of new commodities in the agriculture, the selection of new items in the basket is done on the basis of increased importance in wholesale markets. Varieties, which have declined in importance, need to be dropped in the revised series. Final inclusion or exclusion of an item in the basket is based on the process of consultation with the various departments. The exercise of adding /deleting commodities, specifications and markets is completed once the consultation process is over. In the existing WPI series, items, their specifications and markets have been finalized in consultation of with the Directorate of E&S (M/O Agriculture), National Horticulture Board, Spices Board, Tea board, Coffee Board and Rubber Board, Silk Board, Directorate Of Tobacco, Cotton Corporation of India etc.

4. Derivation of Weighting Diagram

Weights used in the WPI are value weights not quantity weights as its difficult to assign quantity weights. Distribution of the appropriate weight to each of the item is most important exercise for reliable index. Unlike consumer price indices, where weights are derived on the basis of results of Expenditure Surveys, several sources of data are used for derivation of weights for WPI.

5) Collection of Prices

In WPI pricing methodology used is specification pricing. Under this, in consultation with the identified source agencies, precise specifications of all items in the basket are defined for repeat pricing every week. All characteristics like make, model, features along with the unit of sale, type of packaging, if applicable, etc are recorded and printed in the price collection schedule. At the time of scrutiny of price data all these are kept in mind. This pricing to constant quality technique is the cornerstone of Laspeyres formula. In case of changes in quality and specifications, due adjustments are made as per the standard procedures.

The collection of base prices is done concurrently while the work on finalization of index basket is on. Therefore, price collection is normally done for larger number of items pending finalization. Once the basket is ready, current prices are collected only as per the final basket from the designated sources. Weekly prices need to be collected for pre-determined day of the week. For the current series prices are quoted on the basis of the prevailing prices of every Friday. Agricultural wholesale prices are for bulk transactions and include transport cost. Non-agricultural prices are ex-mine or ex-factory inclusive of excise duty but exclusive of rebate if any.

6) Treatment of prices collected from open market & administered prices:

There are some items which constitute part of index baskets but the prices for these items are either totally administered by the Government or are under dual pricing policy. The issue of using administered prices for index compilation is resolved by taking into account appropriate ratio between the levy and non-levy portions. Where these ratios are not available, the issues can be resolved through taking the appropriate number of price quotations of the administered prices and the open market prices after periodic review.

Due to variation in quality and different price movements of the commodities belonging to unorganized sector, separate quotations from organized and unorganized units have to be taken and merged based on the turnover value of both the sectors at item level. For pricing from unorganized sector, adequate number of price quotations has to be drawn out of the list of units by criteria of share of production as far as possible.

7) Classification structure:

The Working Groups over the period have been suggesting to bring the classification of various items under different groups and sub-groups as per the latest revised National Industrial Classification (NIC) which in turn is comparable to International Standard Industrial Classification (ISIC). The classification based on NIC renders the WPI data amenable to comparison with the Index of Industrial Production (IIP) and National Income data.

Major Group/Groups: I. Primary Articles II. Fuel, Power, Light & Lubricants III. Manufactured Products

8) Methodology of Index Calculation

Actual index compilation is done in stages.

In the first stage, once the price data are scrutinized, price relative for each price quote is calculated. Price relative is calculated as the ratio of the current price to the base price multiplied by 100 i.e. $(P_1/P_o) \times 100$.

In the next stage, commodity/item level index is arrived at as the simple arithmetic average of the price relatives of all the varieties (each quote) included under that commodity. An average of price ratio/ relative is used under implicit assumption that each price quotation collected for an item/commodity index compilation has equal importance i.e. the shares of production value is equal.

Next, the indices for the sub groups/groups/ major groups are compiled and the aggregation method is based on Laspeyres formula as below:

I= S (Ii x Wi) / S Wi

Where,

I = Index numbers of wholesale prices of a sub- group/group/ major group/ all commodities

S = represents the summation operation,

Ii = Index of the ith item / sub- group/ group/ major group.

Wi = Weight assigned to the ith item of sub- group/group/ major group.

The weights are value weights. Aggregation is first done at sub-group and group level. All commodities index is compiled by aggregating Major group indices.

9) Handling of the Seasonal Commodities:

There are number of agriculture items, especially some fruits and vegetables, which are of seasonal nature. When a particular seasonal item disappears from the market and its prices are not available because of its being out of season, the weights of such item is imputed amongst the other items on pro rata basis with in the sub-group of vegetables or fruits. The underlying assumption is that if the items remained available, the prices of these items would have moved in the same proportion as the prices of the other items in the sub-group, which did remain available. This is equivalent to giving a greater weight to the remaining items. The seasonality problem can be sorted by adopting other methods like, i) prices of unavailable items can also be extrapolated forward from the period of availability or ii) if such seasonal item has insignificant weight it can be taken permanently from the basket etc.

2. Consumer Price Index Number

The Consumer Price Index (CPI) is a measure of the average change over time in the prices of consumer items -goods and services that people buy for day-to-day living. The CPI is a complex construct that combines economic theory with sampling and other statistical techniques and uses data from several surveys to produce a timely and precise measure of average price change for the consumption sector.

Consumer Price Index is a comprehensive measure used for estimation of price changes in a basket of goods and services representative of consumption expenditure is called consumer price index. The calculation involved in the estimation of CPI is quite rigorous. Various categories and sub-categories have been made for classifying consumption items and on the basis of consumer categories like urban or rural. Based on these indices and sub-indices obtained, the final overall index of price is calculated mostly by national statistical agencies. It is one of the most important statistics for an economy and is generally based on the weighted average of the prices of commodities. It gives an idea of the cost of living.

Inflation is measured using CPI. The percentage change in this index over a period of time gives the amount of inflation over that specific period, i.e. the increase in prices of a representative basket of goods consumed.

The CPI frequently is called a cost-of-living index, but it differs in important ways from a complete cost-of-living measure. A cost-of-living index would measure changes over time in the amount that consumers need to spend to reach a certain utility level or standard of living. Both the CPI and a cost-of-living index would reflect changes in the prices of goods and services, such as food and clothing that are directly purchased in the marketplace; but a complete cost-of-living index would go beyond this role to also take into account changes in other governmental or environmental factors that affect consumers' well-being. It is very difficult to determine the proper treatment of public goods, such as safety and education, and other broad concerns, such as health, water quality, and crime, that would constitute a complete cost-of-living framework.

How do we read or interpret an index?

An index is a tool that simplifies the measurement of movements in a numerical series. Most of the specific CPI indexes have a 1982-84 reference base. That is, the agency computing the index sets the average index level (representing the average price level)-for the 36-month period covering the years 1982, 1983, and 1984-equal to 100. The agency then measures changes in relation to that figure. An index of 110, for example, means there has been a 10-percent increase in price since the reference period; similarly, an index of 90 means a 10-percent decrease. Movements of the index from one date to another can be expressed as changes in index points (simply, the difference between index levels), but it is more useful to express the movements as percent changes. This is because index points are affected by the level of the index in relation to its reference period, while percent changes are not.

	Item A	**Item B**	**Item C**
Year I	112.500	225.000	110.000
Year II	121.500	243.000	128.000
Change in index points	9.000	18.000	18.000
Percent change	9.0/112.500 x 100 = 8.0	18.0/225.000 x 100 = 8.0	18.0/110.000 x 100 = 16.4

In the table above, Item A increased by half as many index points as Item B between Year I and Year II. Yet, because of different starting indexes, both items had the same percent change; that is, prices advanced at the same rate. By contrast, Items B and C show the same change in index points, but the percent change is greater for Item C because of its lower starting index value.

Uses of cost of living index numbers:

1. Cost of living index numbers indicate whether the real wages are rising or falling. In other words they are used for calculating the real wages and to determine the change in the purchasing power

of money.

$$\text{Purchasing power of money} = \frac{1}{\text{Cost of living index number}}$$

$$\text{R}eal\ Wages = \frac{\text{Money wages}}{\text{Cost of living index umbers}} \times 100$$

2. Cost of living indices are used for the regulation of D.A or the grant of bonus to the workers so as to enable them to meet the increased cost of living.
3. Cost of living index numbers are used widely in wage negotiations.
4. These index numbers also used for analysing markets for particular kinds of goods.

Main steps or problems in construction of cost of living index numbers

Production of the CPI requires the skills of many professionals, including economists, statisticians, computer scientists, data collectors, and others.

The cost of living index numbers measures the changes in the level of prices of commodities which directly affects the cost of living of a specified group of persons at a specified place. The general index numbers fails to give an idea on cost of living of different classes of people at different places.

Different classes of people consume different types of commodities, people's consumption habit is also vary from man to man, place to place and class to class i.e. richer class, middle class and poor class. For example the cost of living of rickshaw pullers at BBSR is different from the rickshaw pullers at Kolkata. The consumer price index helps us in determining the effect of rise and fall in prices on different classes of consumers living in different areas.

The following are the main steps in constructing a cost of living index number.

1. **Decision about the class of people for whom the index is meant** It is absolutely essential to decide clearly the class of people for whom the index is meant i.e. whether it relates to industrial workers, teachers, officers, labours, etc. Along with the class of people it is also necessary to decide the geographical area covered by the index, such as a city, or an industrial area or a particular locality in a city.
2. **Conducting family budget enquiry** Once the scope of the index is clearly defined the next step is to conduct a sample family budget enquiry i.e. we select a sample of families from the class of people for whom the index is intended and scrutinize their budgets in detail. The enquiry should be conducted during a normal period i.e. a period free from economic booms or depressions. The purpose of the enquiry is to determine the amount; an average family spends on different items. The family budget enquiry gives information about the nature and quality of the commodities consumed by the people. The commodities are being classified under following heads
i) Food ii) Clothing iii) Fuel and Lighting iv) House rent v) miscellaneous
3. **Collecting retail prices of different commodities** The collection of retail prices is a very important and at the same time very difficult task, because such prices may vary from place to place, shop to shop and person to person. Price quotations should be obtained from the local markets, where the class of people reside or from super bazaars or departmental stores from which they usually make their purchases.

Method of Constructing the Index

The index may be constructed by applying any of the following methods:

1) Aggregate Expenditure Method or Aggregation Method
2) Family Budget Method or the Method of Weighted Relatives.

1. Aggregate Expenditure Method.

When this method is applied the quantities of commodities consumed by the particular group in the base year are estimated and these figures are used as weights. Then the total expenditure on each commodity for each year is calculated.

$$\textbf{Consumer Price Index} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Where

p_1 and p_0 stand for the prices of the current year and base year.

q_1 and q_0 stand for the quantities of the current year and base year.

Steps:

i) The prices of commodities for various groups for the current year is multiplied by the quantities of the base year and their aggregate expenditure of current year is obtained .i.e. $\sum p_1 q_0$

ii) Similarly obtain $\sum p_0 q_0$

iii) The aggregate expenditure of the current year is divided by the aggregate expenditure of the base year and the quotient is multiplied by 100.

Symbolically $\dfrac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$

2. Family Budget Method

When this method is applied the family budgets of a large number of are carefully studied and the aggregate expenditure of the average family on various items is estimated. These values are used as weights.

$$\textbf{Consumer Price Index} = \frac{\sum PV}{\sum v}$$

Where $p = \dfrac{p_1}{p_o} \times 100$ for each item

$v = p_0 q_0$, value on the base year

Example

Construct the Consumer price index number of 2013 on the basis of 2009 from the following data using 1) the aggregate expenditure method and 2) the family budget method.

Commodity	**Quantity in units in 2009**	**Price per unit in 2000 (□)**	**Price per unit in 2013 (□)**
A	100	8	12
B	25	6	7.50

C	10	5	5.25
D	20	48	52
E	25	15	16.50
F	30	9	27

Solution

(1) Aggregate expenditure method

Formula for aggregate expenditure method =

$$Consumer\ Price\ Index = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Commodity	**Price per unit in 2000 (□) : P_0**	**Price per unit in 2013 (□) : P_1**	**Quantity in units in 2009: q_0**	p^0q^0	p^1q^0
A	8	12	100	800	1200
B	6	7.5	25	150	187.5
C	5	5.25	10	50	52.5
D	48	52	20	960	1040
E	15	16.5	25	375	412.5
F	9	27	30	270	810
			Total	$\sum p^0 q^0 = 2605$	$\sum p^1 q^0 = 3702.50$

$$Consumer\ Price\ Index = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

$$= \frac{3702.50}{2605} \times 100 = 142.13$$

2. The family budget method

$$Consumer\ Price\ Index = \frac{\sum PV}{\sum v}$$

Where

$$p = \frac{P_1}{P_0} \times 100$$

for each item

$V = P_0 q_0$, value on the base year

Commodity	**Price per unit in 2000 (□) P_0**	**Price per unit in 2013 (□) P_1**	**Quantity in units in 2009 q_0**	$p = \frac{P1}{P0} \times 100$	$V = P^0 q^0$	PV
A	8	12	100	150	800	120000

B	6	7.5	25	125	150	18750
C	5	5.25	10	105	50	5250
D	48	52	20	108.33	960	104000
E	15	16.5	25	110	375	41250
F	9	27	30	300	270	81000
				898.33	2605	370250

$$Consumer\ Price\ Index = \frac{\sum PV}{\sum v} = \frac{370250}{2605} = 142.13$$

Note: It should be noted that the answer obtained by applying the aggregate expenditure method and family budget method is the same.

Given below is an example of Consumer Price Index for Kerala

Department of Economics & Statistics

No.P3.Pdl.1/2014/DES Thiruvananthapuram , 05/09/2014

Consumer Price Index (Cost of Living Index) Numbers for Agricultural and Industrial Workers for the Month of July 2014

Vide G.O.(MS) No.7/2002/Plg. dated 21-3-2002 of Planning and Economics Affairs (B) Department , Government of Kerala and Government Notification No. G.O (Rt) No 2728/2001/LBR dated 14-09-2001 published in the Kerala Gazette extra ordinary No.1381(Vol. XLVI) dated 15-09-2001.

(Base 1998-99 =100)

Slno.	Centre	Linking Factor*	Index Numbers for		Estimated Indices for	
			June 2014	July 2014	June 2014	July 2014
1	Thiruvananthapuram	10.39	268	273	2785	2836
2	Kollam	10.28	267	271	2745	2786
3	Punalur	9.96	271	275	2699	2739
4	Pathanamthitta	-	284	288	-	-
5	Alappuzha	10.45	272	276	2842	2884
6	Kottayam	10.40	269	272	2798	2829
7	Mundakayam	10.12	276	280	2793	2834
8	Munnar	10.03	250	253	2508	2538
9	Ernakulam	9.92	261	265	2589	2629
10	Chalakkuddy	10.60	267	270	2830	2862
11	Thrissur	10.05	248	251	[illegible]	[illegible]

Possible errors in construction of cost of living index numbers:

Cost of living index numbers or its recently popular name consumer price index numbers are not accurate due to various reasons.

1. Errors may occur in the construction because of inaccurate specification of groups for whom the index is meant.
2. Faulty selection of representative commodities resulting out of unscientific family budget enquiries.
3. Inadequate and unrepresentative nature of price quotations and use of inaccurate weights
4. Frequent changes in demand and prices of the commodity
5. The average family might not be always a representative one.

Wholesale price index numbers (Vs) consumer price index numbers:

1. The wholesale price index number measures the change in price level in a country as a whole. For example economic advisors index numbers of wholesale prices. Where as cost of living index numbers measures the change in the cost of living of a particular class of people stationed at a particular place. In this index number we take retail price of the commodities.
2. The wholesale price index number and the consumer price index numbers are generally different because there is lag between the movement of wholesale prices and the retail prices.
3. The retail prices required for the construction of consumer price index number increased much faster than the wholesale prices i.e. there might be erratic changes in the consumer price index number unlike the wholesale price index numbers.
4. The method of constructing index numbers in general the same for wholesale prices and cost of living. The wholesale price index number is based on different weighting systems and the selection of commodities is also different as compared to cost of living index number

Limitations or demerits of index numbers:

Although index numbers are indispensable tools in economics, business, management etc, they have their limitations and proper care should be taken while interpreting them. Some of the limitations of index numbers are

1. Since index numbers are generally based on a sample, it is not possible to take into account each and every item in the construction of index.
2. At each stage of the construction of index numbers, starting from selection of commodities to the choice of formulae there is a chance of the error being introduced.
3. Index numbers are also special type of averages, since the various averages like mean, median, G.M have their relative limitations, their use may also introduce some error.
4. None of the formulae for the construction of index numbers is exact and contains the so called *formula error*. For example Lasperey's index number has an upward bias while Paasche's index has a downward bias.
5. An index number is used to measure the change for a particular purpose only. Its misuse for other purpose would lead to unreliable conclusions.
6. In the construction of price or quantity index numbers it may not be possible to retain the uniform quality of commodities during the period of investigation.

3. STOCK MARKET INDEX NUMBER

A stock market index is a measure of the relative value of a group of stocks in numerical terms. As the stocks within an index change value, the index value changes. An index is important to measure the performance of investments against a relevant market index.

An Index is used to give information about the price movements of products in the financial, commodities or any other markets. Financial indexes are constructed to measure price movements of stocks, bonds, T-bills and other forms of investments. Stock market indexes are meant to capture the overall behaviour of equity markets. A stock market index is created by selecting a group of stocks that are representative of the whole market or a specified sector or segment of the market. An Index is calculated with reference to a base period and a base index value.

Stock indexes are useful for benchmarking portfolios, for generalizing the experience of all investors, and for determining the market return used in the Capital Asset Pricing Model (CAPM).

A hypothetical portfolio encompassing all possible securities would be too broad to measure, so proxies such as stock indexes have been developed to serve as indicators of the overall market's performance. In addition, specialized indexes have been developed to measure the performance of more specific parts of the market, such as small companies.

It is important to realize that a stock price index by itself does not represent an average return to shareholders. By definition, a stock price index considers only the prices of the underlying stocks and not the dividends paid. Dividends can account for a large percentage of the total investment return.

A stock market index (or just "index) is a number that measures the relative value of a group of stocks. As the stocks in this group change value, the index also changes value. If an index goes up by 1% then that means the total value of the securities which make up the index have gone up by 1% in value.

A stock market index measures the change in the stock prices of the index's components.

How it works/Example:

Let's say we want to measure the performance of the Indian stock market. Assume there are currently four public companies that operate in the United States: Company A, Company B, Company C, and Company D.

In the year 2000, the four companies' stock prices were as follows:

- Company A □ 10
- Company B □ 8
- Company C □ 12
- Company D □ 25
- Total □ 55

To create an index, we simply set the total (□ 55) in the year 2000 equal to 100 and measure any future periods against that total. For example, let's assume that in 2001 the stock prices were:

- Company A □ 4
- Company B □ 38
- Company C □ 12
- Company D □ 24

- Total 78

Because 78 is 41.82% higher than the 2000 base, the index is now at 141.82. Every day, month, year, or other period, the index can be recalculated based on current stock prices.

Note that this index is price-weighted (i.e., the larger the stock price, the more influence it has on the index). Indexes can be weighted by any number of metrics, including shares outstanding, market capitalization, or stock price.

Some Important Stock Market Indices

Symbol	Name
XAX	Amex Composite
VOLNDX	DWS NASDAQ-100 Volatility Target Index
FTSEQ500	FTSE NASDAQ 500 Index
RCMP	NASDAQ Capital Market Composite Index
IXIC	NASDAQ Composite
NQGM	NASDAQ Global Market Composite
NQGS	NASDAQ Global Select Market Composite
QOMX	NASDAQ OMX 100 Index
ILTI	NASDAQ OMX AeA Illinois Tech Index
QMEA	NASDAQ OMX Middle East North Africa Index
IXNDX	NASDAQ-100
NYA	NYSE Composite
OMXB10	OMX Baltic 10
OMXC20	OMX Copenhagen 20
OMXH25	OMX Helsinki 25
OMXN40	OMX Nordic 40
OMXS30	OMX Stockholm 30 Index
RUI	Russell 1000
RUT	Russell 2000
RUA	Russell 3000
OEX	S&P 100
SPX	S&P 500
MID	S&P MidCap
NDXE	The NASDAQ-100 Equal Weighted Index
VINX30	VINX 30
WLX	Wilshire 5000

Types of Stock Market Indices (National Stock Exchange)

(a) Broad Market Indices

These indices are broad-market indices, consisting of the large, liquid stocks listed on the Exchange. They serve as a benchmark for measuring the performance of the stocks or portfolios such as mutual fund investments.

Examples

- CNX Nifty(The CNX Nifty is a well diversified 50 stock index accounting for 23 sectors of the economy. It is used for a variety of purposes such as benchmarking fund portfolios, index based derivatives and index funds.)
- CNX Nifty Junior
- LIX15 Midcap
- CNX 100
- Nifty Midcap 50
- CNX Midcap
- CNX Smallcap Index
- India VIX

(b) Sectoral Indices

Sector-based index are designed to provide a single value for the aggregate performance of a number of companies representing a group of related industries or within a sector of the economy.

Examples

CNX Auto Index (The CNX Auto Index is designed to reflect the behaviour and performance of the Automobiles sector which includes manufacturers of cars & motorcycles, heavy vehicles, auto ancillaries, tyres, etc. The CNX Auto Index comprises of 15 stocks that are listed on the National Stock Exchange.)

CNX Bank Index

CNX Energy Index

CNX Finance Index

CNX FMCG Index

CNX IT Index

CNX Media Index

CNX Metal Index

CNX Pharma Index

CNX PSU Bank Index

CNX Realty Index

IISL CNX Industry Indices

(c) Thematic Indices

Thematic indices are designed to provide a single value for the aggregate performance of a number of companies representing a theme.

Examples

CNX Commodities Index (The CNX Commodities Index is designed to reflect the behaviour and performance of a diversified portfolio of companies representing the commodities segment which includes sectors like Oil, Petroleum Products, Cement, Power, Chemical, Sugar, Metals and Mining. The CNX Commodities Index comprises of 30 companies that are listed on the National Stock Exchange (NSE).)

CNX Consumption Index	CNX Service Sector Index
CPSE Index	CNX Shariah25
CNX Infrastructure Index	CNX Nifty Shariah / CNX 500 Shariah
CNX MNC Index	CNX PSE Index

(d) Strategy Indices

Strategy indices are designed on the basis of quantitative models / investment strategies to provide a single value for the aggregate performance of a number of companies. Strategic indices are designed on the basis of quantitative models / investment strategies to provide a single value for the aggregate performance of a number of companies.

CNX 100 Equal Weight (The CNX 100 Equal Weight Index comprises of same constituents as CNX 100 Index (free float market capitalization based Index).

The CNX 100 tracks the behavior of combined portfolio of two indices viz. CNX Nifty and CNX Nifty Junior. It is a diversified 100 stock index. The maintenance of the CNX Nifty and the CNX Nifty Junior are synchronized so that the two indices will always be disjoint sets; i.e. a stock will never appear in both indices at the same time.)

CNX Alpha Index	CNX Nifty Dividend
CNX Defty	NV20 Index
CNX Dividend Opportunities Index	NI15 Index
CNX High Beta Index	Nifty TR 2X Leverage
CNX Low Volatility Index	Nifty TR 1X Inverse

(e) Fixed Income Indices

Fixed income index is used to measure performance of the bond market. The fixed income indices are useful tool for investors to measure and compare performance of bond portfolio. Fixed income indices also used for introduction of Exchange Traded Funds.

Examples

GSEC10 NSE Index (GSEC10 NSE index is constructed using the prices of top 5 (in terms of traded value) liquid GOI bonds with residual maturity between 8 to 13 years and have outstanding issuance exceeding Rs.5000 crores. The individual bonds are assigned weights considering the traded value and outstanding issuance in the ratio of 40:60.The index measures the changes in the prices of the bond basket.)

GSECBM NSE Index

(f) Index Concepts

Indices and index-linked investment products provide considerable benefits. Important concepts and terminologies are associated with Index construction. These concepts are important for investors to learn from the information that indices contain about investment opportunities.

In the investment world, however, risk is inseparable from performance and, rather than being desirable or undesirable, is simply necessary. Understanding risk is one of the most important parts of a financial education.

Indices and index-linked investment products provide considerable benefits. But it is equally important to know the associated risk that comes as part of such exposure. Important concepts and terminologies are associated with Indices. For e.g. Beta helps us to understand the concepts of passive and active risk. Impact cost represents the cost of executing a transaction in a given stock, for a specific predefined order size, at any given point of time. These concepts are important for to understanding indices and investment opportunities.

(g) Index Funds

An Index Fund is a type of mutual fund with a portfolio constructed to match the constituents of the market index, such as CNX Nifty. An index fund provides broad market exposure and lower operating expenses for investors.

Index Funds today are a source of investment for investors looking at a long term, less risky form of investment. The success of index funds depends on their low volatility and therefore the choice of the index.

Examples

1 Principal Index Fund
2 UTI Nifty Index Fund
3 Franklin India Index Fund
4 SBI Nifty Index Fund
5 ICICI Prudential Index Fund
6 HDFC Index Fund - Nifty Plan
7 Birla Sun Life Index Fund
8 LIC NOMURA MF Index Fund - Nifty Plan

Uses of Stock Market Indices

With any type of investment it's important to measure the performance of that investment. Otherwise there's no way for you to distinguish between a good return on your money versus a bad one.

A relevant stock market index serves that purpose. If your investments consistently lag behind the index then you know you have a poor performer, and it may be time to find a new investment.

Stock market indexes are useful for a variety of reasons. Some of them are :

- They provide a historical comparison of returns on money invested in the stock market against other forms of investments such as gold or debt.
- They can be used as a standard against which to compare the performance of an equity fund.
- In It is a lead indicator of the performance of the overall economy or a sector of the economy

- Stock indexes reflect highly up to date information
- Modern financial applications such as Index Funds, Index Futures, Index Options play an important role in financial investments and risk management

BSE SENSEX (Bombay Stock Exchange Sensitive Index)

The Sensex is an "index". What is an index? An index is basically an indicator. It gives you a general idea about whether most of the stocks have gone up or most of the stocks have gone down. The Sensex is an indicator of all the major companies of the BSE.

BSE SENSEX is considered as the Barometer of Indian Capital Markets. If the Sensex goes up, it means that the prices of the stocks of most of the major companies on the BSE have gone up. If the Sensex goes down, this tells you that the stock price of most of the major stocks on the BSE have gone down.

BSE SENSEX, first compiled in 1986, was calculated on a "Market Capitalization-Weighted" methodology of 30 component stocks representing large, well-established and financially sound companies across key sectors. The base year of S&P BSE SENSEX was taken as 1978-79. S&P BSE SENSEX today is widely reported in both domestic and international markets through print as well as electronic media. It is scientifically designed and is based on globally accepted construction and review methodology. Since September 1, 2003, BSE SENSEX is being calculated on a free-float market capitalization methodology. The "free-float market capitalization-weighted" methodology is a widely followed index construction methodology on which majority of global equity indices are based; all major index providers like MSCI, FTSE, STOXX, and Dow Jones use the free-float methodology.

The BSE Sensex currently consists of the following 30 major Indian companies as of October 2014

Axis Bank Ltd	ITC Ltd
Bajaj Auto Ltd	Larsen & Toubro Ltd
Bharat Heavy Electricals Ltd	Mahindra and Mahindra Ltd
Bharti Airtel Ltd	Maruti Suzuki India Ltd
Cipla Ltd	NTPC Ltd
Coal India Ltd	Oil and Natural Gas Corporation Ltd
Dr.Reddy's Laboratories Ltd	Reliance Industries Ltd
GAIL (India) Ltd	Sesa Goa Ltd
HDFC Bank Ltd	State Bank of India
Hero MotoCorp Ltd	Sun Pharmaceutical Industries Ltd
Hindalco Industries Ltd	Tata Consultancy Services Ltd
Hindustan Unilever Ltd	Tata Motors Ltd
Housing Development Finance Corporation Ltd	Tata Power Company Ltd
ICICI Bank Ltd	Tata Steel Ltd
Infosys Ltd	Wipro Ltd

Nifty (National Stock Exchange Index)

Just like the Sensex which was introduced by the Bombay stock exchange, Nifty is a major stock index in India introduced by the National stock exchange.

NIFTY was coined fro the two words 'National' and 'FIFTY'. The word fifty is used because; the index consists of 50 actively traded stocks from various sectors.

So the nifty index is a bit broader than the Sensex which is constructed using 30 actively traded stocks in the BSE.

Nifty is calculated using the same methodology adopted by the BSE in calculating the Sensex – but with three differences. They are:

- The base year is taken as 1995
- The base value is set to 1000
- Nifty is calculated on 50 stocks actively traded in the NSE
- 50 top stocks are selected from 24 sectors.

The selection criteria for the 50 stocks are also similar to the methodology adopted by the Bombay stock exchange.

Nifty, is a weighted average of 50 stocks, meaning some stocks hold more "value" than other stocks. For example ITC has more weight than Lupin.

List of 50 stocks that have been included in the nifty as on October 2014.

Name	Sector
ACC Ltd.	CEMENT AND CEMENT PRODUCTS
Ambuja Cements Ltd.	CEMENT AND CEMENT PRODUCTS
Asian Paints Ltd.	PAINTS
Axis Bank Ltd.	BANKS
Bajaj Auto Ltd.	AUTOMOBILES - 2 AND 3 WHEELERS
Bank of Baroda	BANKS
Bharat Heavy Electricals Ltd.	ELECTRICAL EQUIPMENT
Bharat Petroleum Corporation Ltd.	REFINERIES
Bharti Airtel Ltd.	TELECOMMUNICATION - SERVICES
Cairn India Ltd.	OIL EXPLORATION/PRODUCTION
Cipla Ltd.	PHARMACEUTICALS
Coal India Ltd	MINING
DLF Ltd.	CONSTRUCTION
Dr. Reddy's Laboratories Ltd.	PHARMACEUTICALS
GAIL (India) Ltd.	GAS
Grasim Industries Ltd.	CEMENT AND CEMENT PRODUCTS
HCL Technologies Ltd.	COMPUTERS - SOFTWARE
HDFC Bank Ltd.	BANKS
Hero Honda Motors Ltd.	AUTOMOBILES - 2 AND 3 WHEELERS
Hindalco Industries Ltd.	ALUMINIUM

Hindustan Unilever Ltd.	PERSONAL CARE
Housing Development Finance Corporation Ltd.	FINANCE - HOUSING
I T C Ltd.	CIGARETTES
ICICI Bank Ltd.	BANKS
IndusInd Bank Ltd.	BANKS
Infosys Technologies Ltd.	COMPUTERS - SOFTWARE
Infrastructure Development Finance Co. Ltd.	FINANCIAL INSTITUTION
Jindal Steel & Power Ltd.	STEEL AND STEEL PRODUCTS
Kotak Mahindra Bank Ltd.	BANKS
Larsen & Toubro Ltd.	ENGINEERING
Lupin Ltd.	PHARMACEUTICALS
Mahindra & Mahindra Ltd.	AUTOMOBILES - 4 WHEELERS
Maruti Suzuki India Ltd.	AUTOMOBILES - 4 WHEELERS
NMDC Ltd.	MINING
NTPC Ltd.	POWER
Oil & Natural Gas Corporation Ltd.	OIL EXPLORATION/PRODUCTION
Power Grid Corporation of India Ltd.	POWER
Punjab National Bank	BANKS
Reliance Industries Ltd.	REFINERIES
Sesa Sterlite Ltd.	MINING
State Bank of India	BANKS
Sun Pharmaceutical Industries Ltd.	PHARMACEUTICALS
Tata Consultancy Services Ltd.	COMPUTERS - SOFTWARE
Tata Motors Ltd.	AUTOMOBILES - 4 WHEELERS
Tata Power Co. Ltd.	POWER
Tata Steel Ltd.	STEEL AND STEEL PRODUCTS
Tech Mahindra Ltd.	COMPUTERS - SOFTWARE
UltraTech Cement Ltd.	CEMENT AND CEMENT PRODUCTS
United Spirits Ltd.	BREW/DISTILLERIES
Wipro Ltd.	COMPUTERS - SOFTWARE

Nifty and the Sensex

The Sensex and Nifty are both Indices. The Sensex, also called the BSE 30, is a stock market index of 30 well-established and financially sound companies listed on Bombay Stock Exchange (BSE). The Nifty, similarly, is an indicator of the 50 top major companies on the National Stock Exchange (NSE).

The Sensex and Nifty are both indicators of market movement. If the Sensex or Nifty go up, it means that most of the stocks in India went up during the given period. If the Nifty goes down, this tells you that the stock price of most of the major stocks on the BSE have gone down.

Just in case you are confused, the BSE, is the Bombay Stock Exchange and the NSE is the National Stock Exchange. The BSE is situated at Bombay and the NSE is situated at Delhi. These are the major stock exchanges in the country. There are other stock exchanges like the

Calcutta Stock Exchange etc. but they are not as popular as the BSE and the NSE.Most of the stock trading in the country is done though the BSE & the NSE.

Uses of Index numbers in Economics

Economists frequently use index numbers when making comparisons between sets of data across time. For example, a macroeconomist might want to measure changes in the cost of living in India over a five-year period. This is where index numbers come in; they allow for easy, quick comparisons by identifying a 'base year' and scaling all of the other results off of that year.

Economists frequently use index numbers when making comparisons over time. Index numbers are used to measures all types of quantitative changes in the agricultural, industrial, and commercial fields, as also in such economic magnitudes as income, employment, exports, imports, prices, etc. A close study of such changes helps the government to adopt appropriate monetary and fiscal measures in order to achieve growth with stability.

Some of the uses of index numbers are discussed below:

Index numbers possess much practical importance in measuring changes in the cost of living, production trends, trade, income variations, etc.

1. In measuring changes in the value of money:

Index numbers are used to measure changes in the value of money. A study of the rise or fall in the value of money is essential for determining the direction of production and employment to facilitate future payments and to know changes in the real income of different groups of people at different places and times. As pointed out by Crowther, "By using the technical device of an index number, it is thus possible to measure changes in different aspects of the value of money, each particular aspect being relevant to a different purpose."

2. In cost of living:

Cost of living index numbers in the case of different groups of workers throw light on the rise or fall in the real income of workers. It is on the basis of the study of the cost of living index that money wages are determined and dearness and other allowances are granted to workers. The cost of living index is also the basis of wage negotiations and wage contracts.

3. In analyzing markets for Goods and Services:

Consumer price index numbers are used in analyzing markets for particular kinds of goods and services. The weights assigned to different commodities like food, clothing, fuel, and lighting, house rent, etc., govern the market for such goods and services.

4. In measuring changes in Industrial Production:

Index numbers of industrial production measure increase or decrease in industrial production in a given year as compared to the base year. We can know from such as index number the actual condition of different industries, whether production is increasing or decreasing in them, for an industrial index number measures changes in the quantity of production.

5. In internal Trade:

The study of indices of the wholesale prices of consumer and industrial goods and of industrial production helps commerce and industry in expanding or decreasing internal trade.

6. In external Trade:

The foreign trade position of a country can be accessed on the basis of its export and import indices. These indices reveal whether the external trade of the country is increasing or decreasing.

7. In economic policies:

Index numbers are helpful to the state in formulating and adopting appropriate economic policies. Index numbers measure changes in such magnitudes as prices, incomes, wages, production, employment, products, exports, imports, etc. By comparing the index numbers of these magnitudes for different periods, the government can know the present trend of economic activity and accordingly adopt price policy, foreign trade policy and general economic policies.

8. In determining the foreign exchange rate:

Index numbers of wholesale price of two countries are used to determine their rate of foreign exchange. They are the basis of the purchasing power parity theory which determines the exchange rate between two countries on inconvertible paper standard.

Summary of Index Numbers

Index numbers are statistical measures designed to show changes in a variable or group of related variables with respect to time, geographic location or other characteristics such as income, profession, etc. A collection of index numbers for different years, locations, etc., is sometimes called an index series.

Index numbers are commonly used statistical device for measuring the combined fluctuations in a group related variables. If we wish to compare the price level of consumer items today with that prevalent ten years ago, we are not interested in comparing the prices of only one item, but in comparing some sort of average price levels. We may wish to compare the present agricultural production or industrial production with that at the time of independence. Here again, we have to consider all items of production and each item may have undergone a different fractional increase (or even a decrease). How do we obtain a composite measure? This composite measure is provided by index numbers which may be defined as a device for combining the variations that have come in group of related variables over a period of time, with a view to obtain a figure that represents the ‘net’ result of the change in the constitute variables.

Index numbers may be classified in terms of the variables that they are intended to measure. In business, different groups of variables in the measurement of which index number techniques are commonly used are (i) price, (ii) quantity, (iii) value and (iv) business activity (Eg. Sensex). Thus, we have index of wholesale prices, index of consumer prices, index of industrial output, index of value of exports and index of business activity, etc. Here we shall be mainly interested in index numbers of prices showing changes with respect to time, although methods described can be applied to other cases. In general, the present level of prices is compared with the level of prices in the past. The present period is called the current period and some period in the past is called the base period.

Simple Index Number:

A simple index number is a number that measures a relative change in a single variable with respect to a base.

Composite Index Number:

A composite index number is a number that measures an average relative changes in a group of relative variables with respect to a base.

Types of Index Numbers:

Following types of index numbers are usually used:

Price index Numbers:

Price index numbers measure the relative changes in prices of a commodities between two periods. Prices can be either retail or wholesale.

Quantity Index Numbers:

These index numbers are considered to measure changes in the physical quantity of goods produced, consumed or sold of an item or a group of items.

TIME SERIES ANALYSIS

A time series is simply a sequence of numbers collected at regular intervals over a period of time. Technically speaking a Time Series is an ordered sequence of values of a variable at equally spaced time intervals. In statistics, a time series is a sequence of numerical data points in successive order, usually occurring in uniform intervals. This concerns the analysis of data collected over time, such as weekly values, monthly values, quarterly values, yearly values, etc.

Many statistical methods relate to data which are independent, or at least uncorrelated. There are many practical situations where data might be correlated. This is particularly so where repeated observations on a given system are made sequentially in time. Data gathered sequentially in time are called a time series.

Here are some examples in which time series arise:

- Economics and Finance
- Environmental Modelling
- Meteorology and Hydrology
- Demographics
- Medicine
- Engineering
- Quality Control

The simplest form of data is a longish series of continuous measurements at equally spaced time points. That is observations are made at distinct points in time, these time points being

equally spaced and, the observations may take values from a continuous distribution.

The above setup could be easily generalized: for example, the times of observation need not be equally spaced in time; the observations may only take values from a discrete distribution.

If we repeatedly observe a given system at regular time intervals, it is very likely that the observations we make will be correlated. So we cannot assume that the data constitute a random sample. The time-order in which the observations are made is vital.

Objectives of time series analysis:

- Description - summary statistics, graphs

• Analysis and interpretation - find a model to describe the time dependence in the data, can we interpret the model

• Forecasting or prediction - given a sample from the series, forecast the next value, or the next few values

• Control - adjust various control parameters to make the series fit closer to a target

• Adjustment - in a linear model the errors could form a time series of correlated observations, and we might want to adjust estimated variances to allow for this

Types of time Series

1. Continuous
2. Discrete

Discrete means that observations are recorded in discrete times. It says nothing about the nature of the observed variable. The time intervals can be annually, quarterly, monthly, weekly, daily, hourly etc.

Continuous means that observations are recorded continuously -e.g. temperature and/or humidity in some laboratory. Again, time series can be continuous regardless of the nature of the observed variable.

Discrete time series can result when continuous time series are sampled. Sometimes quantities that don't have an instantaneous value get aggregated also resulting in a discrete time series e.g. daily rainfall we will mostly study discrete time series in this course. Note that discrete time series are often the result of discretization of continuous time series (e.g. monthly rainfall).

Uses of time series

There are two main uses of time series analysis: (a) identifying the nature of the phenomenon represented by the sequence of observations, and (b) forecasting (predicting future values of the time series variable). Both of these goals require that the pattern of observed time series data is identified and more or less formally described. Once the pattern is established, we can interpret and integrate it with other data (i.e., use it in our theory of the investigated phenomenon, e.g., seasonal commodity prices). Regardless of the depth of our understanding and the validity of our interpretation (theory) of the phenomenon, we can extrapolate the identified pattern to predict future events.

The usage of time series models is twofold:

- Obtain an understanding of the underlying forces and structure that produced the observed data
- Fit a model and proceed to forecasting, monitoring or even feedback and feed forward control.

Time Series Analysis is used for many applications such as:

- Economic Forecasting
- Sales Forecasting
- Budgetary Analysis
- Stock Market Analysis
- Yield Projections
- Process and Quality Control
- Inventory Studies

- Workload Projections
- Utility Studies
- Census Analysis

Time series analysis can be useful to see how a given asset, security or economic variable changes over time or how it changes compared to other variables over the same time period. For example, in stock market investments, suppose you wanted to analyze a time series of daily closing stock prices for a given stock over a period of one year. You would obtain a list of all the closing prices for the stock over each day for the past year and list them in chronological order. This would be a one-year, daily closing price time series for the stock. Delving a bit deeper, you might be interested to know if a given stock's time series shows any seasonality, meaning it goes through peaks and valleys at regular times each year. Or you might want to know how a stock's share price changes as an economic variable, such as the unemployment rate, changes.

The analysis of time series is of great significance not only to the economists and business man but also to the scientist, astronomist, geologist etc. for the reasons given below.

1) It helps in understanding past behavior. It helps to understand what changes have taken place in the past. Such analysis is helpful in predicting the future behavior.
2) It helps in planning future operations: Statistical techniques have been evolved which enable time series to be analyzed in such a way that the influence which has determined the form of that series may be ascertained. If the regularity of occurrence of any feature over a sufficient long period could be clearly established then. Within limits, prediction of probable future variations would become possible.
3) It helps in evaluating current accomplishments. The actual performance can be compared with the expected performance and the cause of variation analyzed. For example, if expected sale for 2000-01 was 10,000 washing machines and the actual sale was only 9000. One can investigate the cause for the shortfall in achievement.
4) It facilitates comparison. Different time series are often compared and important conclusions drawn there from.

Components of Time Series

The fluctuations of time series can be classified into four basic type of variations, they are often called components or elements of a time series. They are:

(1) Secular Trend or Long Term Movements (T)
(2) Seasonal Variations (S)
(3) Cyclical Variations (C)
(4) Irregular Variations (I)

The value (y) of a phenomenon observed at any point of time (t) is the net effect of all the above mentioned categories of components of a time series. We will see them in detail here.

(1) Secular Trend

The secular trend is the main component of a time series which results from long term effect of socio-economic and political factors. This trend may show the growth or decline in a time series over a long period. This is the type of tendency which continues to persist for a very long period. Prices, export and imports data, for example, reflect obviously increasing tendencies over time.

(2) Seasonal Variations (Seasonal Trend)

These are short term movements occurring in a data due to seasonal factors. The short term is generally considered as a period in which changes occur in a time series with variations in weather or festivities. For example, it is commonly observed that the consumption of ice-cream during summer us generally high and hence sales of an ice-cream dealer would be higher in some months of the year while relatively lower during winter months. Employment, output, export etc. are subjected to change due to variation in weather. Similarly sales of garments, umbrella, greeting cards and fire-work are subjected to large variation during festivals like Onam, Eid, Christmas, New Year etc. These types of variation in a time series are isolated only when the series is provided biannually, quarterly or monthly.

(3) Cyclical Variations (Cyclical Variations)

These are long term oscillation occurring in a time series. These oscillations are mostly observed in economics data and the periods of such oscillations are generally extended from five to twelve years or more. These oscillations are associated to the well-known business cycles. These cyclic movements can be studied provided a long series of measurements, free from irregular fluctuations is available.

(4) Irregular Variations (Irregular Fluctuations)

These are sudden changes occurring in a time series which are unlikely to be repeated, it id that component of a time series which cannot be explained by trend, seasonal or cyclic movements .It is because of this fact these variations some-times called residual or random component. These variations though accidental in nature, can cause a continual change in the trend, seasonal and cyclical oscillations during the forthcoming period. Floods, fires, earthquakes, revolutions, epidemics and strikes etc,. are the root cause of such irregularities.

Measurement of Trend: Moving Average and the Method of least squares:

Mean of time series data (observations equally spaced in time) from several consecutive periods. Called 'moving' because it is continually recomputed as new data becomes available, it progresses by dropping the earliest value and adding the latest value. For example, the moving average of six-month sales may be computed by taking the average of sales from January to June, then the average of sales from February to July, then of March to August, and so on. Moving averages (1) reduce the effect of temporary variations in data, (2) improve the 'fit' of data to a line (a process called 'smoothing') to show the data's trend more clearly, and (3) highlight any value above or below the trend.

1. Method of Moving Averages

Let us explain the concept of Moving Average with the aid of an example.

Suppose that the demand for skilled laborers for a construction project is given for the last 7 months as shown in the following table:

Month	Demand
1	120
2	110
3	90
4	115
5	125
6	117
7	121

The engineer who is in charge of this project needs to predict the demand for the next month (the 8th month) based on the available data. He decided to take the average of the data and predicted the demand as follows.

Average = (120 + 110 + 90 + 115 + 125 + 117 + 121)/7 = 114

But this method has a disadvantage. The above method is known as the Simple Mean Forecasting Method. The main problem with this method is the space limitation for storing all of the past data. If the data contains several thousand items, each of which has several hundred data records, you need a lot of memory space to store this data on your computer. In addition, this method is not very sensitive to a shift in recent data if it contains a large number of data points.

A solution to these problems is the Moving Averages technique. Using this method, you need to maintain only the N most recent periods of data points. At the end of each period, the oldest period's data is discarded and the newest period's data is added to the data base. The average is then divided by N and used as a forecast for the next period.

The formula for a three period moving average is given below:

$$Three\ period\ moving\ average\ =\ MA(3)\ =\ M_{t+1} = \frac{[D_t + D_{t-1} + D_{t-2}]}{3}$$

Now using the three period moving average, the average for the above problem can be calculated as follows.

$$=\ MA(3)\ =\ M_{7+1} = \frac{[D_7 + D_6 + D_5]}{3} = \frac{125 + 117 + 121}{3} = 121$$

So from the above example we can summarize as follows.

When a trend is to be determined by the method of moving average value for a number of years is secured and this average is taken as the normal or trend value for the unit of time falling at the middle of the period covered in the calculation of the average. While applying this method, it is necessary to select a period for moving average such as 3 yearly, 5 yearly or 8 yearly moving average etc.

The 3 yearly moving averages shall be computed as follows:

$$\frac{a+b+c}{3},\quad \frac{b+c+d}{3},\ \frac{c+d+e}{3},\ \frac{d+e+f}{3}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

5 yearly moving average

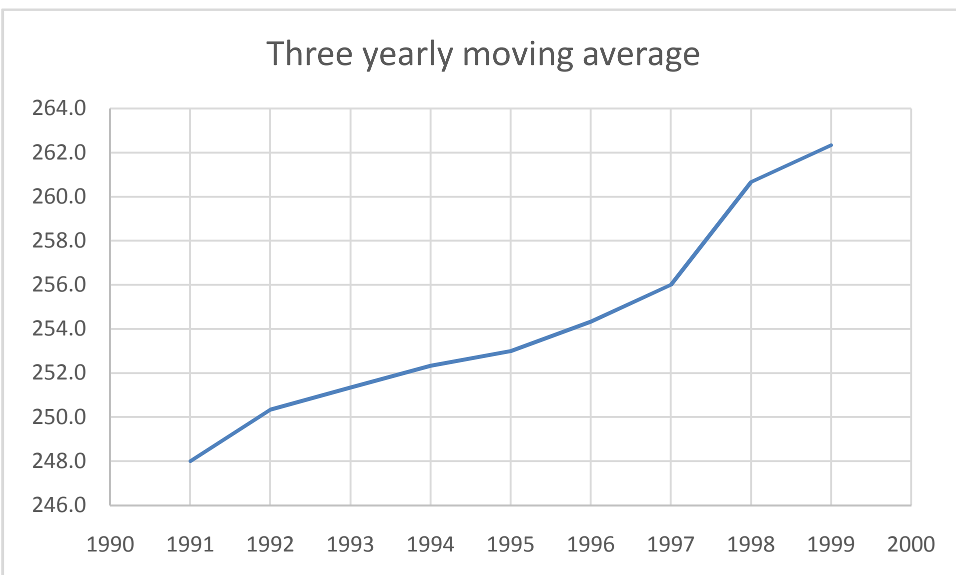
$$\frac{a+b+c+d+e}{5},\frac{b+c+d+e+f}{3},\ \frac{c+d+e+f+g}{3}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

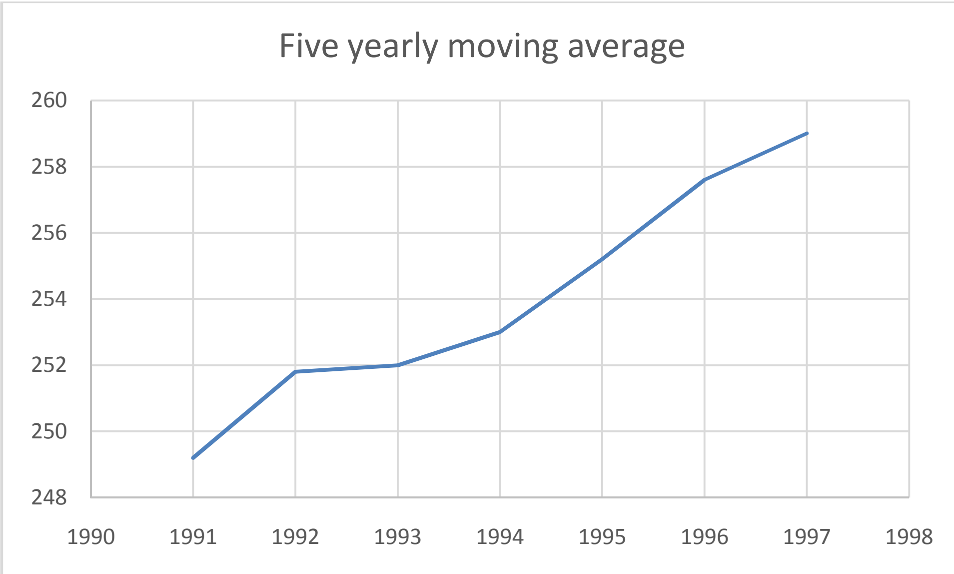
Example: Calculate the 3 yearly moving average and 5yearly moving average of the producing figures given below.

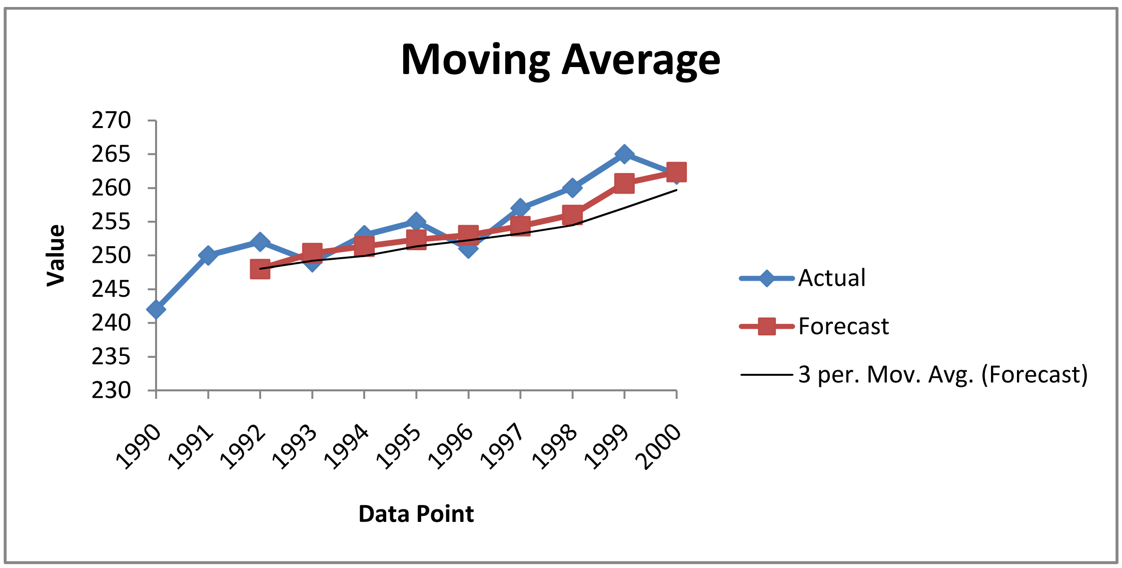
For computing three yearly trend, first find three yearly moving totals a+b+c, b+c+d, c+d+e etc (Column 3 in the following table). Then find average of each. Since it is sum of three observations, divide each by 3 to get average. $\frac{a+b+c}{3}$, $\frac{b+c+d}{3}$, $\frac{c+d+e}{3}$ $,etc.$ Repeat the same process for 5 years taking 5 instead of 3.

Year	Y	3 yearly moving totals	3 yearly moving	5 yearly moving	5 yearly moving

(1)	(2)	(3)	averages (trend values) (4) =(3)÷3	totals (5)	averages (trend values) (6) = (5) ÷ 5
1990	242	_	_	_	_
1991	250	744	248.0	1246	249.2
1992	252	751	250.3	1259	251.8
1993	249	754	251.3	1260	252
1994	253	757	252.3	1265	253
1995	255	759	253.0	1276	255.2
1996	251	763	254.3	1288	257.6
1997	257	768	256.0	1295	259
1998	260	782	260.7	_	_
1999	265	787	262.3	_	_
2000	262	_	_	_	_







Merits of Moving Average Method

- It is simple as compared to the method of least squares.
- It is flexible, if a few more figures are added to the data; the entire calculations are not changed.
- It has the advantage that it follows the general movements of the data and that its shape is determined by the data rather than statisticians choice of a mathematical function.
- It is particularly effective if the trend of a series is very irregular.

Limitations:

- Trend values cannot be computed for all the years. The moving averages for the first few years and last few years cannot be obtained. It is often these extreme years in which h we may be interested.

- Selection of proper period is a great difficulty. If a wrong period is selected, there is ever likelihood that conclusions may be misleading.
- Since the moving average is not represented by a mathematical function, this method cannot be used for forecasting.
- It can be applied only to those series which show periodically.

2. METHOD OF LEAST SQUARES:

Least Squares Method is a statistical technique to determine the line of best fit for a model. The least squares method is specified by an equation with certain parameters to observed data. This method is extensively used in regression analysis and estimation.

In the most common application - linear or ordinary least squares - a straight line is sought to be fitted through a number of points to minimize the sum of the squares of the distances (hence the name "least squares") from the points to this line of best fit.

In contrast to a linear problem, a non-linear least squares problem has no closed solution and is generally solved by iteration. The earliest description of the least squares method was by Carl Freidrich Gauss in 1795.

Field data is often accompanied by noise. Even though all control parameters (independent variables) remain constant, the resultant outcomes (dependent variables) vary. A process of quantitatively estimating the trend of the outcomes, also known as regression or curve fitting, therefore becomes necessary.

The curve fitting process fits equations of approximating curves to the raw field data. Nevertheless, for a given set of data, the fitting curves of a given type are generally NOT unique. Thus, a curve with a minimal deviation from all data points is desired. This best-fitting curve can be obtained by the method of least squares.

The principle of least squares provides us an analytical or mathematical device to obtain an objective fit to the trend of the given time series. Most of the data relating to economic and business time series conform to definite laws of growth or predictions. This technique can be used to fit linear as well as nonlinear trends.

Fitting linear trend

A straight line can be fitted to the data by the method of curve fitting based on the most popular principle called the principle of least squares. Such a straight line is also known as Line of Best fit. Let the line of best fit be described by an equation of the type $y = a+bx$ where y is the value of dependent variable, a and b are two unknown constants whose values are to be determined.

To find a and b, we apply the method of least squares. Let 'E' be the sum of the squares of the deviations of all the original values from their respective values derived from the equations. So that $E = [y - (a+bx)]^2$

By Calculus method, for minimum $\frac{\partial E}{\partial b} = 0$. Thus we get the two equations known as Normal equations. They are:

$$\sum y = na + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

Solving these two normal equations, we get a and b. Substituting these values in the equation y = a+bx, we get the trend equation.

Example:

Fit a linear trend to the following data by the least square method.

Year	2000	2002	2004	2006	2008
Production	18	21	23	27	16

Solution

Let x = t -2004 ….(I)

Let the trend line of y (production) on x be

$y = a + bx, \ (Origin\ 2004)$ …..(II)

Year (t)	y	x=t-2004	x^2	xy	Ye=21+0.1x	Y-Ye
2000	18	-4	16	-72	20.6	-2.6
2002	21	-2	4	-42	20.8	0.2
2004	23	0	0	0	21	2
2006	27	2	4	54	21.2	5.8
2008	16	4	16	64	21.4	-5.4
	$\sum y = 105$	$\sum x = 0$	$\sum x^2 = 40$	$\sum xy = 4$		$\sum (y - y_e) = 0$

The normal equations for estimating and b in (II) are

$$\sum y = na + b \sum x \quad and \quad \sum xy = a \sum x + b \sum x^2$$

$$105 = 5a + b \times 0 \qquad 4 = a \times 0 + b \times 40$$

$$a = \frac{105}{5} = 21 \qquad b = \frac{4}{40} = \frac{1}{10} = 0.1$$

Substituting in (II), the straight line trend equation is given by
Y = 21+0.1x, (Origin :2004) ……..(III)
[x units = 1 year and y = production in '000 units)]
Putting $x = -4, -2, 0, 2$ and 4 in (III), we obtain the trend values (y_e) for the years 2000, 2002…2008 respectively, as given in last but one column of the table above.
The difference $(y - y_e)$ is calculated in the last column of the table.
We have

$$\sum (y - y_e) = -2.6 + 0.2 + 2.0 + 5.8 - 5.4 = 8 - 8 = 0, \textit{as required.}$$

Uses of Method of Least Squares

The least square methods (LSM)is probably the most popular technique in statistics. This is due to several factors.

First, most common estimators can be casted within this framework. For example, the mean of a distribution is the value that minimizes the sum of squared deviations of the scores.

Second, using squares makes LSM mathematically very tractable because the Pythagorean theorem indicates that, when the error is independent of an estimated quantity, one can add the squared error and the squared estimated quantity.

Third, the mathematical tools and algorithms involved in LSM (for eg. derivatives) have been well studied for a relatively long time.

The use of LSM in a modern statistical framework can be traced to Galton (1886) who used it in his work on the heritability of size which laid down the foundations of correlation and (also gave the name to) regression analysis. The two antagonistic giants of statistics Pearson and Fisher, who did so much in the early development of statistics, used and developed it in different contexts (factor analysis for Pearson and experimental design for Fisher).

Nowadays, the least square method is widely used to find or estimate the numerical values of the parameters to fit a function to a set of data and to characterize the statistical properties of estimates. It exists with several variations: Its simpler version is called ordinary least squares(OLS), a more sophisticated version is called weighted least squares (WLS), which often performs better than OLS because it can modulate the importance of each observation in the final solution. Recent variations of the least square method are alternating least squares (ALS) and partial least squares (PLS).

Problems with least squares

Despite its popularity and versatility, LSM has its problems. Probably, the most important drawback of LSM is its high sensitivity to outliers (i.e., extreme observations). This is a consequence of using squares because squaring exaggerates the magnitude of differences (e.g., the difference between 20 and 10 is equal to 10 but the difference between 20 2and 102 is equal to 300) and therefore gives a much stronger importance to extreme observations. This problem is addressed by using robust techniques which are less sensitive to the effect of outliers. This field is currently under development and is likely to become more important in the next future.

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Module III
FUNDAMENTALS OF PROBABILITY

We are very familiar with the word probability because in our daily life we use the concept of probability. In different situations we use the probability. For example there is a 50-50 chance to win the game. This chapter deals with the concept of probability. It discusses the meaning of probability, types of probability, calculation of probability etc…

First we have to discuss some of the concepts of set theory which are necessary for probability theory

1-Set

A Set is a collection of well defined objects. For example, cards in a packet of cards, Odd numbers from 1 to 13, members of cricket team of college etc Normally, we specify a set by listing its members or elements in parentheses { }.

For example $A = \{1,3,5,7\}$ means that *A* is the set consisting of numbers 1,3,5,7.

We could also write $\{\text{odd numbers less than 9}\}$

2-Finite Set

A Set which contains a finite or a fixed number of elements is called a Finite Set. For example, even number from 2 to 20

3-Infinite set

A Set which contains an infinite number of elements is called an Infinite Set. For example, set of natural numbers

4-Union of Set

If A and B are two sets, then the union of sets A and B is the set of all elements which belong to either A or B or both. It is denoted by $A \cup B$ .For example,

Let $A = \{1,2,3,4,5\}$ $B = \{4,5,6\}$, then $A \cup B = \{1,2,3,4,5,6\}$

5- Intersection of Set

If A and B are two sets, then the Intersection of sets A and B is the set of all elements which are common to both of them. Intersection of sets A and B is denoted by $A \cap B$.

For example,

Let $A = \{1,2,3,4,5\}$ $B = \{4,5,6\}$, then $A \cap B = \{4,5\}$

6- Disjoint Sets

Two sets are said to be disjoint if they do not have any common element between them.

For example,

Let $A = \{a,b,c\}$ $B = \{d,e,f\}$, here, there no common elements in both sets. So these sets are disjoint sets

Meaning of Probability:

Probability is a number associated with the occurrence of an event in the uncertain cases. In statistics, probability is a numerical value that measures the uncertainty that a particular event will occur. Following are the different definitions of probability.

1. Classical definition
2. Empirical definition
3. Axiomatic definition
4. Subjective definition

In the probability theory we use different terms such as trial, event, experiment etc. We discuss each term in order to understand the probability theory very well.

A- Experiment

An experiment is a process that, when performed, result in one of many observations

B- Random Experiment

An experiment is called random experiment if the exact outcome of the trials of the experiment is unpredictable. It must satisfies the following conditions

1. It must have several possible outcomes
2. It must be repeatable under uniform conditions
3. The outcome must be unpredictable

For example: Tossing a coin

C- Trial

It is an attempt to produce an outcome of a random experiment. For example: I f we toss a coin we are performing trials..

D- Sample Space

The set of all possible outcomes of the given experiment is called the sample space. If a die is tossed, the possible outcomes are 1, 2, 3, 4, 5, 6, so the sample space is the set $\{1,2,3,4,5,6\}$.

A particular outcome (an element) of this set is called sample point

E- Event

The outcomes in an experiment are called event. In other words, it is a set of outcomes of the sample space. Simply it is the sub-set of sample space. For example: If we throw a die, we get 1 or 2 or 3 or 4 or 5 or 6

F- Simple Event and Composite Event

Each of the final outcomes for an experiment is called simple event. This is also called an elementary event. A compound event is collection of more than one outcome for an experiment. This is also called compound event

G- Equally Likely Events

Events are said to be equally likely when we have no reason to expect one rather the other. For example: if a fair coin is tossed, there are two equally likely outcomes: heads (H) or tails (T).

E -Exhaustive Event

The set of all possible outcomes in trial is known as exhaustive cases. For example, in the case of tossing of a fair coin head and tail are exhaustive events

F-Mutually Exclusive Events

. Events that cannot occur together are said to be mutually exclusive events. For example, consider the following two events in a rolling of a normal die

$A = \{2,4,6\}$, $B = \{1,3,5\}$. Here, both events are mutually exclusive

G-Independent Events

Two events are said to be independent, if the probability of the occurrence of one event will not affect the occurrence or nonoccurrence of the second events. Independent events are those events whose probabilities are in no way affected by the occurrence of any other event proceeding, following or occurring at the same time.

In other words, A and B are independent events if

Either $P(A|B) = P(A)$ or $P(B|A) = P(B)$

H-Dependent Events

Two events are said to be dependent if the occurrence or non-occurrence of one event in any trial affects the probability of other events in other trials. Thus in the case of dependent events, the probability of any event is conditional, or depends upon the occurrence or non occurrence of other events.

I. Complementary Events

The complements of event A, denoted by , $\bar{A}$ is the event that includes all the outcomes for an experiment that are not in A.

For example, in a group of 175 people, 125peoples drink cola at least once. If one person is selected randomly, what is the probability that that the selected person drinks cola and what is the probability that that the selected person does not use cola

Solution: Let A= person drinks cola

$\bar{A}$ = person does not use cola

P(A)= 125/175= 0.71 P(B)= 50/175=0.29

1- Classical Definition (A prior Probability)

It assumes that the outcomes of a random experiment are equally likely. It defines probability as "Probability is the ratio of the numbers of favourable cases to the total number of equally likely events".

That is

$$P(A) \quad \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

Here P (A) denotes the probability of occurrence of A

P(A) ranges from 0 to 1

For example, find the probability of obtaining a head in a tossing of fair coin

Solution: The two outcomes, head and tail are equally likely out comes. Therefore

$$P(head)\frac{\text{Number of favourablecases}}{\text{Total number of equally likely cases}}$$

$$=\frac{1}{\text{Total number of equally likely cases}}=\frac{1}{2}=0.50$$

2- Relative Frequency Theory of Probability

It can be defined as the relative frequency with which an event occurs in an indefinitely large number of trials.

If an event occurs m times out of n trials, its relative frequency is $\frac{m}{n}$, The value which is approached by $\frac{m}{n}$ when 'n' trials tends to infinity is called the probability of the event.

Symbolically, $P(A)=\lim_{n\to\infty}\frac{m}{n}$

For example: Suppose Mr. Ali randomly selects 1500nfamilies from the Malabar region and he found that 800 of them own homes. What is the probability of the family who owns home?

Solution: Here, we have two outcomes, 'family own home' and 'family does not own home'. These two events are not equally likely. Hence, the classical probability cannot be applied. Here, we use relative frequency approach

m=800, n=1500

$$P(\text{a randomly selected family owns a home})=\frac{800}{1500}=0.53$$

Remember, if every time Ali repeats this experiment he may get different probability for this event.

3-Subjective Approach of Probability

This was introduced by Frank Ramsey in 1926. The subjective probability may be defined as the probability assigned to an event by an individual based on whatever evidence available. For example, if an individual wants to find out the probability of electing Mr. John from one constituency, he may assign a value between 0 to 1 according to his belief for possible occurrence, by taking into various consideration such as his past performance, political affiliation, opponent party etc.

4-Axiomatic Approach

This theory was introduced by A.N. Kolmogorov in the year 1933. As per this approach no precise definition of probability is given, rather certain axioms or postulates on which probability calculations are based are given.

(1) The probability of an event ranges from 0 to 1. If the event cannot take place its probability shall be '0' and if it occur its probability is '1'.

(2) The probability of the entire sample space is i.e., P(S) =1.

(3) If A and B are mutually exclusive events then the probability of occurrence of either A or B denoted by P ($A \cup B$) shall be given by –

$$P(A \cup B) = P(A) + P(B).$$

Conditional Probability

The multiplication theorem cannot applicable if the events are dependent. The probability attached to such an event is called the conditional probability, and is denoted by P(A/B) i.e., probability of A, given that 'B' has occurred. It is defined as;

$$P(A|B)=\frac{P(A\cap B)}{P(B)},P(B)\neq 0$$

If two events 'A' and 'B' are dependent, then conditional probability of 'B' given 'A' is −

$$P(B\setminus A)=\frac{P(A\cap B)}{P(A)},P(A)\neq 0$$

For example, A survey is conducted in a college, 150 students of this college were asked whether they are in favour of or against increasing the working hours of college. The responses are given below.

Student	In Favour	Against	Total
Boy	35	55	90

Girl	20	40	60
	55	95	150

Compute the conditional probability $P(\text{In favour}|girl)$

Solution: This is the conditional probability that randomly selected student is in favour given that this student is girl. it is known that the event girl is already occurred. So we concerned only with the second row of the table

The conditional probability is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Here event A is infavour and event B is girl. Hence

$$P(\text{In favour}|girl) = \frac{\text{Number of girls who are in favour}}{\text{Total number of girls}} = \frac{20}{60} = 0.33$$

Marginal Probability

Marginal probability is the probability of a single event without consideration of any other event. It is also called simple probability.

For example, A group consists of 200 peoples, out of these 120 are males and 80 are females. What is the probability that a male is selected?

Solution: the probability that a male will be selected is obtained by dividing the total of the male by the total people

$$P(male) = \frac{\text{Number of males}}{\text{Total number of employees}} = \frac{120}{200} = 0.6$$

Similarly,

$$P(female) = \frac{\text{Number of females}}{\text{Total number of employees}} = \frac{80}{200} = 0.4$$

Addition Theory of Probability

This is used to calculate the probability of the union of events. The union of events A and B is the collection of all outcomes that belongs either event A or event B or to both A and B

This states that if two events A and B are mutually exclusive, the probability of occurrence A or B is the sum of the individual probability of A and B. Symbolically,

P (A or B) = P (A) + P (B)

OR

P (A $\cup$ B) = P (A) + P (B)

Here $\cup$ denotes union, it is the set of all elements belongs to either A or B or both

Similarly If A and B are any two events, the probability for the occurrence of either A or B both is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Theorem of Probability

Multiplication is used to find the probability of two or more events happening together.

A- Multiplication Rule to Find Joint probability

The probability of the intersection of two events is called joint probability. It is written as P(A and B). It is also written as $P(A \cap B|)$ or P(*AB)*

The probability of the intersection of two independent events A and B is

P(A and B) = P(A) $P(B|A)$

For example, The following table gives the classification of all legislative members by gender and Law graduate

Gender	Law Graduate (L)	Not a Law Graduate (N)	Total
Male(M)	18	32	50
Female(F)	14	26	40
	32	58	90

If one of the MLA is selected random, what is the probability that this member is male and Law Graduate?

Solution: W calculate the probability of intersection of two events, M and L

$$P(\text{M and L}) = P(M)P(L|M)$$

P(M)= 50/90 and $P(L|M) = 18/50$

$$P(\text{M and L}) = P(M)P(L|M) = \frac{50}{90} \times \frac{18}{50} = 0.36$$

Note: The joint probability of two mutually exclusive events is always Zero. If A and B is two mutually exclusive events, then,

P(A and B) = 0

B- Multiplication Rule to Find Probability of Independent Events

If two events A and B are independent, the probability that they both will occur is equal to the product of their individual probabilities. Symbolically,

$$P(AandB) = P(A)P(B)$$

For example: A fair coin is tossed four times. What is the probability that all the four tosses are tail?

A- Let events are A, B, C, and D

Then P(A)=P(B)=P(C)=P(D)= $\frac{1}{2}$, In each events tail has $\frac{1}{2}$ probability

Since A, B, C and d are independent the probability of four tails;

$$P(A \cap B \cap C \cap D) = P(A) \times P(B) \times P(C) \times P(D) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

Bayes' Theorem

We use Bayes Theorem for revising a probability value based on additional information that is later obtained. It is the extension of conditional probability. If A and B denote two events, P(A|B) denotes the conditional probability of A occurring, given that B occurs. The two conditional probabilities P(A|B) and P(B|A) are in general different. Bayes theorem gives a relation between P(A|B) and P(B|A).

Here, we deal with the repeated events whereby new additional information is obtained for a subsequent event, and that new information is used to revise the probability of the initial event. As a result we will have two probability values; prior probability and posterior probability

A prior probability is an initial probability value originally obtained before any additional information is obtained. A posterior probability is a probability value that has been revised by using additional information that is later obtained.

Theorem

The probability of event *A,* given that event *B* has subsequently occurred, is

$$P(A|B) = \frac{P(A).P(B|A)}{\left[P(A).P(B|A).P(\bar{A}).P(B|\bar{A}) \right]}$$

For example, in a city 57% peoples are males and 43% peoples are females. Later, it is found that 12% of males drink alcohol and 3.5% of females drink alcohol

a- Find the prior probability that the selected person is a male.

b- By using the additional information to find the probability that the selected subject is a male.

Solution

Let's use the following notation

M = male $\bar{M}$ = female (or not male)

D = drinks alcohol $\bar{D}$= not a drunkard a- P(M)=0.57= male $(\bar{M})$ = 0.43

$$P(D|M) = 0.125, \quad P(D|\bar{M}) = 0.035$$

b- $$P(M|D) = \frac{P(M).P(D|M)}{\left[P(M).P(D|M).P(\bar{M}).P(D|\bar{M})\right]}$$

$$P(M|D) = \frac{0.57 * 0.125}{[0.57 * 0.125 + 0.43 + 0.035]} = 0.54$$

Permutation

Any arrangement of a set n objects in a given order is called a permutation of the objects. We use the notation nP_r .Here, letter P stands for permutation. It is also denoted as P(n,r). To fin d the value of this expression we use factorials. That is;

$$nP_r = \frac{n!}{(n - r!)}$$

Suppose we want to arrange 4 books ,say W, X, Y,Z by choosing 2 of these books at a time.

The arrangement of 4 books by taking two books at a time is denoted by $4P_2 = \frac{4!}{(4-2)!} = \frac{4.3.2.1}{2.1} = 12$

They are WX, XY,YZ,, XW, YX, ZY, WY, WZ,YW, ZW, XZ, ZX

Combination

A combination is a grouping or a collection of all or a part of a given number of things without reference to their order of arrangement. In other words, a combination of n objects taken r at a time is any selection of r o these objects where order does not count. The number of combinations of n objects taken r ata time will be denoted by nC_r or C(n,r)

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r!(n - r)!}$$

QUESTIONS

1. What do you mean by random experiment?

Answer: An experiment is called random experiment if the exact outcome of the trials of the experiment is unpredictable. It must satisfies the following conditions

a. It must have several possible outcomes
b. It must be repeatable under uniform conditions
c. The outcome must be unpredictable

For example: Tossing a coin

2. Define event

Answer: The outcomes in an experiment are called event. For example: If we throw a die, we get 1 or 2 or 3 or 4 or 5 or 6

3. Define exhaustive event

Answer: The total number of possible events in any trial is known as exhaustive events**.** For example, in a tossing of fair coin exhaustive events are two, head and tail

4. Define equally likely events

Answer: Two events are considered to be equally likely if one of them cannot be expected in preference to the other. For example, in a tossing of a fair coin there is equal chance to get head or tail. There no preference to occur head.

5. Define mutually exclusive

Answer: Events are said to be mutually exclusive if the happening of any one of them excludes the happening of all others in a trial. In other words, Events that cannot occur together are said to be mutually exclusive events. For example: In a tossing of fair coin the events of turning head or tail are mutually exclusive.

6. Define independent Events

Answer: Two events are said to be independent if the occurrence of one does not affect the probability of the occurrence of the other In other words, A and B are independent events if

Either $P(A|B) = P(A)$ or $P(B|A) = P(B)$

7. Define Complementary Events

Answer: The complements of event A, denoted by , $\bar{A}$ is the event that includes all the outcomes for an experiment that are not in A.

8. State the classical definition of probability

Answer: It defines probability as "Probability is the ratio of the numbers of favourable cases to the total number of equally likely events".

That is;

$$P(A) \quad \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

Here P (A) denotes the probability of occurrence of A

P(A) ranges from 0 to

$$P(head) \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

$$= \frac{1}{\text{Total number of equally likely cases}} = \frac{1}{2} = 0.50$$

9. State Relative Frequency Theory of Probability

It can be defined as the relative frequency with which an event occurs in an indefinitely large number of trials. If an event occurs m times out of n trials, its relative frequency is $\frac{m}{n}$, The value which is approached by $\frac{m}{n}$ when 'n' trials tends to infinity is called the probability of the event.

Symbolically, $P(A) = \lim_{n \to \infty} \frac{m}{n}$

10. What do you mean by Subjective Approach of Probability

Answer: The subjective probability may be defined as the probability assigned to an event by an individual based on whatever evidence available. For example, if an individual wants to find out the probability of electing Mr. John from one constituency, he may assign a value between 0 to 1 according to his belief for possible occurrence, by taking into various consideration such as his past performance, political affiliation, opponent party etc.

11. What is the meaning of Independent Events

Answer: Two events are said to be independent, if the probability of the occurrence of one event will not affect the occurrence or nonoccurrence of the second events. Independent events are those events whose probabilities are in no way affected by the occurrence of any other event proceeding, following or occurring at the same time

12. Define dependent Events

Answer: Two events are said to be dependent if the occurrence or non-occurrence of one event in any trial affects the probability of other events in other trials. Thus in the case of dependent events, the probability of any event is conditional, or depends upon the occurrence or non occurrence of other events.

13. State the multiplication theorem for dependent events.

Answer: If A and B are any two events

$$P(A \cap B) = P(A)P(B/A) or P(A \cap B) = P(B)P(A/B)$$

14. Satate the multiplication theorem for independent Events

Answer: If A and B are two independent events

$$P(A \cap B) = P(A)P(B)$$

If A, B and C are independent, then $P(A \cap B \cap C) = P(A)P(B)P(C)$

14. State Axiomatic Approach of Probability theory

Answer: Axiomatic Approach of Probability theory was introduced by A.N. Kolmogorov in the year 1933. As per this approach no precise definition of probability is given, rather certain axioms or postulates on which probability calculations are based are given.

(4) The probability of an event ranges from 0 to 1. If the event cannot take place its probability shall be '0' and if it occur its probability is '1'.

(5) The probability of the entire sample space is i.e., P(S) =1.

(6) If A and B are mutually exclusive events then the probability of occurrence of either A or B denoted by P (A ∪ B) shall be given by –

P (A ∪ B) = P(A) + P (B).

15. What do you mean by Conditional probability

Answer: Conditional probability, is denoted by P(A/B) i.e., probability of A, given that 'B' has occurred. It is defined as;

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

If two events 'A' and 'B' are dependent, then conditional probability of 'B' given 'A' is −

$$P(B \setminus A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$$

16. Define Marginal Probability

Answer: Marginal probability is the probability of a single event without consideration of any other event. It is also called simple probability.

For example, A group consists of 200 peoples, out of these 120 are males and 80 are females. What is the probability that a male is selected?

Solution: the probability that a male will be selected is obtained by dividing the total of the male by the total people

$$P(male) = \frac{\text{Number of males}}{\text{Total number of employees}} = \frac{120}{200} = 0.6$$

17. State Addition Theory of Probability

Answer: This state that if two events A and B are mutually exclusive, the probability of occurrence A or B is the sum of the individual probability of A and B. Symbolically,

P (A or B) = P (A) + P (B)

Similarly If A and B are any two events, the probability for the occurrence of either A or B both is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

18. State multiplication rule to find joint probability

The probability of the intersection of two events is called joint probability. It is written as P(A and B). It is also written as $P(A \cap B|)$ or P(*AB)*

The probability of the intersection of two independent events A and B is

P(A and B) = P(A) $P(B|A)$

19. State multiplication rule to find probability of independent events

Answer: If two events A and B are independent, the probability that they both will occur is equal to the product of their individual probabilities. Symbolically,

$$P(AandB) = P(A)P(B)$$

20. Find the probability of obtaining an odd number in one roll of a die

Answer: The experiment has a total of six outcomes: 1, 2, 3,4, 5, 6.All these outcomes are equally likely. Let B an event that an odd number appear. Event B includes three outcomes: $\{1,3,5\}$ .If any one of these three numbers is obtained, event B is to occu,r hence,

$$P(B)\frac{\text{Number of outcomes included in B}}{\text{Total number of outcomes}}=\frac{3}{6}=\frac{1}{2}=0.5$$

21. In a group of 150 boys 60 have played cricket at least once. Suppose one of these 150 boys randomly selected. What is the probability that he has played cricket at least once?

Answer: Because the selection is to be made randomly, each of the 150 boys has the same probability of being selected. Consequently this experiment has a total of 150 equally outcomes. 60 of these 150 outcomes are included in the event that the selected boy has played cricket at least once

$$P(\text{selected boy has played cricket at least once})\frac{60}{150}=0.4$$

22. Write the sample space of tossing of a coin once

Ans: The sample space S consists of two outcomes, “heads” (H) and “tails” (T). Thus S = {H, T}

23. What is the probability of rolling two coins and getting Head first and then Tail?

Answer: Here $S=\{HH,TT,HT,TH\}$

$$P(H \ \& \ thenT=\frac{1}{4}$$

24. A packet consists of 4 red cards, 3 yellow cards, 2 green cards, and 3 blue card. Suppose an individual randomly select a card from these cards, what is the probability that he will pick red?

Answer: Number of red cards= 4

Total cards= 12

$$P(\mathrm{Re}\,d)=\frac{4}{12}=\frac{1}{3}=0.33$$

25. Mr.X is rolling a die. What is the probability of getting greater than 5?

Here $S=\{1,2,3,4,5,6\}$

We have only one number which is greater than 5

S0, $P(Greatertha\,n5)=\frac{1}{6}=0.17$

26. A bag contains five white and four red balls. Find probability of drawing a red ball

Answer: Here total number of ways in which a ball can be drawn = 9

Ways favourable for drawing a red ball = 4

Hence the probability of drawing a red ball = $P(\mathrm{Re}\,d)=\frac{4}{9}=0.44$

27. A box contains 15 papers which are numbered from 1 to 15.A paper is drawn random, find the probability that the number is

a- even, b-less than 7, c-odd, d- even and less than 7, d- even or less than 7

Answer:

Here, $S=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$

a- There are 7 even numbers, 2,4,6,8,10,12,14; hence $P(Even)=\frac{7}{15}$

b- There are 6 numbers less than 7, 1,2,3,4,5,6;hence $P=\frac{6}{15}$

c- There are odd numbers, 1,3,5,7,9,11,13; hence $P=\frac{7}{15}$

d- There are three numbers,2,4,6, which are even and less than 7 hence $P=\frac{3}{15}$

e- By addition rule $P=\frac{7}{15}+\frac{6}{15}+\frac{7}{15}-\frac{3}{15}=\frac{17}{15}$

28. A card is drawn from an ordinary pack of 52 cards. What is the probability to get spade or an ace?

Answer: Ways in which a card can be spade or an ace= 13+3

The probability= $\frac{16}{52}$

29. A bag contains nine white, six red, and four blue balls. If two balls are drawn at random from the bag, find the probability that both the balls are white.

Answer: The total number of balls 9+ 6+4=19

Total number of ways of drawing 2 white balls out of 19= $19C_2=\frac{19\times 18}{2\times 1}=171$

(a) Ways of drawing 2 white balls out of 9 balls= $9C_2 = \frac{9 \times 8}{2 \times 1} = 36$

Hence the probability of drawing both the white balls

$$\frac{\text{Favourable ways of drawing}}{\text{Total number of ways}} = \frac{36}{171} = 0.21$$

30. Suppose we have two boxes and each boxes contains paper which is numbered from 1 to 8.If P_1 denotes the probability that the sum of the numbers be 8 and P_2 denotes the probability that the sum of the numbers be 10

Answer: Each box contains 8 numbers. Hence total number of ways of choosing one from each=

$$8C_1 \times 8C_1 = 64$$

Now $S_1 = \{(2,6)(3,5)(7,1)(4,4)\}$

$$P_1 = \frac{4}{64} = \frac{1}{16}$$

$$S_2 = \{(2,8)(5,5)(7,3)(6,4)\}$$

$$P_2 = \frac{4}{64} = \frac{1}{16}$$

$$P_1 + P_2 = \frac{1}{16} + \frac{1}{16} = \frac{2}{16}$$

31. What are the demerits of classical approach to the probability theory

Answer: Classical approach defines probability as "Probability is the ratio of the numbers of favourable cases to the total number of equally likely events".

That is;

$$P(A) \quad \frac{\text{Number of favourable cases}}{\text{Total number of equally likely cases}}$$

Here P (A) denotes the probability of occurrence of A

Demerits

(i) In the classical priori definition of probability only equally likely cases are taken into consideration. If the events cannot be considered equally likely classical definition fails to give a good account of the concept of probability.

(ii) When the total number of possible outcomes 'n' becomes infinite, this definition fails to give a measure for probability.

(iii) If we are deviating from the games of chances like, tossing a coin, throwing a die etc. this definition cannot be applied.

31. What re the importance of probability

Answer: Probability is a numerical measure of the likelihood that a spefic event will occur. Following the importance of probability

(1) It helps to solve the social, economic, political and business problems.
(2) Probability has become an indispensible tool for all types of formal studies that involve uncertainty.
(3) Probability theory provides a media of coping up with the uncertainties and forecast the future more accurately may it be related to scientific investigations or day to day life.
(4) It is concerned with problems ranging from the construction of econometric models, managerial decisions on planning and control to occurrence of accidents and random disturbances in an electrical mechanism.

32. State the properties of probability

Answer: Probability is a numerical measure of the likelihood that a specific event will occur. Following are the importance of probability. The properties of probability are

1- The probability of an event always lies in the range 0 to1
For sure event it is 1 and for an impossible event it is 0

The sum of the probabilities of all simple events for an experiment is a

MODULE IV

VITAL STATISTICS

VITAL STATISTICS: MEANING AND USES- FERTILITY RATES: CRUDE BIRTH RATE, GENERAL FERTILITY RATE, SPECIFIC FERTILITY RATE, GROSS REPRODUCTION RATE AND NET REPRODUCTION RATE - MORTALITY RATES: CRUDE DEATH RATE, SPECIFIC DEATH RATE, STANDARDISED DEATH RATE, INFANT MORTALITY RATE AND MATERNAL MORTALITY RATE-SEX RATIO AND COUPLE PROTECTION RATIO.

Meaning

Vital statistics forms the most important branch of statistics and it deals with mankind in the aggregate. It is the science of numbers applied to the life history of communities and nations. Thus, it may refer to a government database (government records) recording the births and deaths of individuals within that government's jurisdiction. The term signifies either the data or the methods applied in the analysis of the data which provide a description of the vital events occurring in given communities. By vital events we mean such events of human life as birth, death, sickness, marriage, divorce, adoption, legitimation, recognition, separation. etc. In short, all the events which have to do with an individual's entrance into or departure from life together with the changes in civil status which may occur to him during his lifetime may be called vital statistics. Thus, the system of counting births, marriages, migration, diseases and disabilities and death is known as vital statistics. Therefore, vital statistics signifies the data or methods applied in the analysis of data pertaining to vital events occurring in a community. Vital statistics have to do with people rather than things. The population census is the most fundamental statistical inquiry provides a picture of the population and its characteristics at one moment of time: Hence, vital statistics provide the tools for measuring the changes which continuously occur in a community or country.

Definitions of Vital Statistics

Vital statistics have been variously defined. Some of the important definitions of Vital Statistics are:

1. According to Arthur Newsholme, Vital Statics may be interpreted in two ways - in a broader sense and in a narrow sense. In a broader sense, it refers to all types of population statistics by whatever mode collected. In a narrower sense it refers only to the statistics derived from registrations of births, deaths and marriages.
2. According to B.Benjamin, "Vital Statistics are conventionally numerical records of marriage, births, sickness, and deaths by which the health and growth of a community may be studied".
3. According to Arthur Newsholme, Vital Statistics is "That branch of biometry which deals with data and the laws of human mortality, morbidity and demography".

A statistical study of human population has two aspects. 1. A study of the composition of the population at a point of time and 2. A study of the changes that occur during a given period, i.e. growth or decline of the population. Changes in the population are the outcome of the events like births, deaths, migration, marriages, divorces, etc., called vital events. Vital statistics is the application of statistical methods to the study of those facts and has been defined as the registration, preparation, transcription, collection, compilation and preservation of data pertaining to the dynamics of the population.

Thus, it is clear from the above definitions that in a broader sense, vital statistics refers to all types of population statistics. The purpose of such statistics is to find out the changing composition of communities (nations) with reference to sex, age, education, birth and death rates, marriage, economics and civic status, etc.

METHODS OF OBTAINING VITAL STATISTICS (Sources of Vital Statistics)

There are three methods of obtaining vital statistics and they are: 1. Registration Method, 2. Census Enumeration and 3. Analytical Method. We will explain the estimation of vital rates using census data

1. **Registration Method:**

The registration method is the most important source of obtaining vital statistics. It may be defined as the continuous and permanent recording of the occurrence of vital events pertaining to births, deaths, marriages, migration etc. These data have their values as legal documents and they are useful as a source of statistics. In most countries there is a system of registering the occurrence of every important vital event under legal requirements. For example, when a child is born, the matter has to be reported to the proper authorities, together with such information as the age of mother, religion of parents, etc. similarly, when a man dies, the death is to be recorded with appropriate authorities and a certificate is to be obtained before the body is cremated.

Continuous permanent recording of vital events can best be ensured by means of legislation which makes registration compulsory. Such legislation should also provide sanctions for the enforcement of the obligation. Thus it will be seen that the registration method is characterized not only by the continuous character of the observations but also by the compulsory nature of the method. Both provisions are fundamental, Registration of vital events for legal purposes is an almost universal requirement. Data on births and deaths can also be obtained from the hospital record.

2. **Census Enumeration:** In most countries of the world population census is undertaken generally at ten years interval. A census is an enumeration at a specified time of individuals inhabiting a specified area, during which particulars are collected regarding age, sex, marital status, occupation, religion, etc. The fundamental deficiency of the census method for collecting vital statistics is that it can, at best, produce returns for the census year and no other. Census years are usually ten years apart. For the intercensal years, current vital statistics are not produced by the census method, and thus, that method fails in the first and minimum requisite for vital statistics, i.e., the production of data on a current basis. Not only does the census method fail to provide intercensal data but it fails also to record completely the occurrence of births and deaths even for the census year. Periodic surveys have been employed to secure ad hoc information on births and deaths in areas where the registration method has not been established or where it is very defective. In such situations, survey has the distinct advantages of making available some vital statistics not otherwise obtainable and of securing at the same time the corresponding population.

3. **Analytical Method: Estimation of Vital Rates using Census Data**. It is assumed that the derivation of birth, death and marriage rates is the object of collecting vital statistics. Hence, there is another method which could be employed to yield these basic facts. This method is mathematical one based on an analysis of the returns of two consecutive censuses of population. The census returns employed must of necessity be the result of very accurate and dependable enumerations, which have produced reliable age and marital status distributions of the population. If certain assumptions are made regarding migration and the reliability of the enumeration is ensured, data from censuses of population can be used to derive information on the approximate

numbers of births. Deaths and marriages which have occurred in the population over the intercensal period. This indirect method yields aggregates only and that too solely for the year of the census. It does not, therefore, justify, its consideration as a method of developing vital statistics which by definition must be current and continuous. However, it is a method which has been developed for estimating vital statistics in Brazil, for example and as such should be mentioned for its applicability to the relatively rare areas which have non-existent or deficient registration statistics but a reliable census of population.

In the following discussion, we shall be concerned with births and deaths – the two most important vital events. It will be assumed that we have from census data for the given community the total size of the population and also its distribution with respect to different characters (i.e. age and sex) corresponding to different points of time: while from registers we have data regarding the number of births and deaths occurring during different periods.

In order to determine the population at a time (say, t) subsequent to a census or between two censuses one may use a number of procedures. A very common method is to make use of statistics of births, deaths, immigration and emigration. The population P1 at time t is then obtained as

$$P_t = P\theta + (B-D) + (I-E)$$

Where P1 = total population at a point of time

$P\theta$ = total population recorded at last census

B = total number of births during the given period

D =-total number of deaths during the given period

I = total number of immigrants

E = total number of emigrants

Uses of Vital Statistics (Importance of Vital Statistics)

Vital statistics are highly useful and they are useful to individuals, operating agencies, in research, demographic and medical field, in public administration and internationally.

1) **Use to individuals**: Records of births, deaths, marriages and divorce etc. are highly useful to the individuals. The basic registration document has legal significance to the person concerned.

2) **Use to operating Agencies:** Records of birth, deaths and marriages are useful to governmental agencies for a variety of administrative purpose. For example, the control programmes for infectious diseases within the family and community often depend on the death registration report for their initiation. Public health programmes of post-natal care for the mother and child usually have their starting point to the birth register and the corresponding birth indices. Public safety, accident prevention, and crime eradication programmes make use of the death registration records.

3) **Use in Research:** Vital statistics are indispensable in demographic research. The study of population movement and of the interrelationships of demographic with economic and social factors is of fundamental importance to society. and will become increasingly more so as the advances in technology and public health focus attention on demographic problems. The three directions which such an analysis takes are: 1) population estimation (2) population projection, and

(3) analytical studies. Very closely allied to the role of vital statistics in demographic research is their use by the medical profession engaged in research. Medical and pharmaceutical research, like demographic research, requires a certain number of guideposts. This guidance may be found in part at least in mortality and natality statistics.

4. **Use in Public Administration:** Vital statistics are, vital to public health. As vital statistics include information on births, deaths and a lot of other health information which are highly useful to public health officials. The health officials must have data on the prevalence of disease and major health issues. Most national governments by law mandate the collection of vital statistics. Thus vital statistics are fundamental elements in public administration, which is the machinery and methods underlying all official programmes of economic and social development in either 'developed' or 'under-developed" areas. The role of vital statistics in overall planning and evaluation of economic and social development is the most important use to which this body of data may be placed. To monitor current demographic trends and action programmes, for scientific research to study the interrelationship between fertility and mortality trends and socio-economic development, vital statistics are indispensable.

5. International use of vital Statistics: Vital statistics are also useful from the international view point. Only by a sufficiently wide survey of human facts can the required norms of all sorts be established, norms which represent the character of the great unit constituted by the aggregation of all the nations.

6. For policy making: Vital statistics form the basis for policy guidance, planning and projections.

However, it should be noted that vital statistics, like all statistics and multiple vital records, are not ends in themselves but tools for the study and understanding of other phenomena.

FERTILITY

The term fertility refers to the actual production of children. In the measurement of growth of population of a country, fertility has an important place. Population of a country may go on changing and changes in the population are due to the changes in births and deaths. Thus, the growth of population in a country is the net result of obtained as the difference between total births and total deaths.

MEASUREMENT OF FERTILITY

Fertility rates vary according to the level of development achieved by a country or region. Generally; developed countries have a much lower fertility rate due to greater wealth, education, and urbanization. Mortality rates are low, birth control is understood and easily accessible, and costs are often deemed very high because of education, clothing, feeding, and social amenities. With wealth, contraception becomes affordable. However, in countries like Iran where contraception was subsidised before the economy accelerated, birth rate also rapidly declined. Further, longer periods of time spent getting higher education often mean women have children later in life. The result is the demographic-economic paradox. Female labor participation rate also has substantial negative impact on fertility. However, this effect is neutralized among Nordic or liberalist countries. In undeveloped countries on the other hand, families desire children for their labour and as caregivers for their parents in old age. Fertility rates are also higher due to the lack of access to contraceptives, generally lower levels of female education, and lower rates of female employment in industry. In order to study the speed at which the population is increasing, fertility rates are used which are of various types. Important amongst these are:

Crude Birth Rate (Birth Rate)

It is the simplest method of measuring fertility. The crude birth rate, computed as the ratio of the number of births to the total population, is more affected by population differences in age and sex ratio. Therefore the crude birth rate is a better measure of tax burden and other economic statistics than the general fertility rate. The crude birth rate may be measured as the number of births in a given population during a given time period (such as a calendar year), divided by the total population and multiplied by 1,000. According to the United Nation's World Population Prospects of the 2008 Revision Population Datab*ase*, the crude birth rate is the number of births over a given period, divided by the person-years lived by the population over that period. It is expressed as the number of births per 1,000 population. It acts as population through child-birth. In this method the number of births is related to the total population. Since it is only a live birth that signifies an addition to the existing population, live births alone are considered in measuring fertility, thus excluding still births.

The annual crude birth rate is defined as:

$$\text{Crude Birth Rate} = \frac{\textbf{Annual Births}}{\textbf{Annual mean population}}\ X\ 1000$$

In this measure the births are related to the mean population and not to the population at a particular date. The crude birth rate of a given year tells us at what rate births have augmented the population over the course of the year.

The crude birth rate usually lies between 10 and 55 per 1.000. The level of the crude birth rate is determined by:

i) The sex and age distribution of the population; and
ii) The fertility of the population. i.e., the average rate of child-bearing of females.

A relatively high crude birth rate can be recorded if the sex and age distribution is favourable even though fertility is low. Thus, countries with a relatively large proportion of population in the 15-50 years age group will have a relatively high crude birth rate, other things being equal. Hence, crude birth rate is not a suitable measure of fertility as the fertility differs from one age group to another. Therefore, we have the fertility rates like i) General Fertility Rate ii) Specific Fertility Rate and iii) Total Fertility Rate.

General Fertility Rate

The General Fertility Rate (GFR) is the birth rate of women of child bearing age (age 15-44). While births to women less than 15 or more than 44 years are included in the general fertility rate, the population for those ages are not. This rate refers to the proportion of the number of children born per 1.000 of females, the reproductive or child-bearing age. Thus the numerator of this rate would remain the same as the crude rate, but the denominator would be limited to the age-sex group of the population able to contribute to the birth rate. The general fertility rate is calculated by dividing the total number of births in a given year by the number of women aged 15 through 44 and multiplying by 1,000. The formula for such a rate is:

$$\text{G.F.R} = \frac{\begin{array}{c}\text{Number of live births which occured among the}\\ \text{Population of a given geographic area during}\\ \text{a given year}\end{array}}{\begin{array}{c}\text{Mid−year female population of ages 15 to 49}\\ \text{in the given geographic area during the same year}\end{array}}\ X\ 1000$$

The computation of the F.G.R. requires that a decision be taken before hand as to which years of the life a woman should be included in the child-bearing period. Although the practice varies in this respect, generally the child-bearing age is taken 15 to 50 years. Births to mothers under 15 and above 50 are so rare that they are not recorded separately but are included in the age-group 15 and 49 respectively.

The G.F.R. shows how much the women in child-bearing ages have added to the existing population through births. It takes into account the sex composition of the population and also the age composition to a certain extent. Yet it is calculated without proper regard to the age composition of the female population in child-bearing ages. The fecundity of women differs according to age-groups. In our country it is low in the age group 15-19 after which it gradually declines. In U.S.A. fecundity reaches its peak in the age-group 20-24 and thereafter declines. For this reason even if the general fertility rate of two populations may correspond to each other, we cannot assume that the fertility rate is really identical unless different age-groups are also taken into consideration in its calculation. It should be noted that the calculation of the general fertility rate is limited solely to live births. It is not a pregnancy rate and does not include induced abortions, fetal deaths (stillbirths), or spontaneous abortions miscarriages). The general fertility rate is the best overall indicator of reproductive behavior and success.

Specific Fertility Rate

Age-specific fertility rate refers to the number of births to females in a particular age category in a particular year compared to the number of females in that age category. **Age-specific fertility rate** is usually expressed as births per woman or births per 1,000 women in the age category. It is usually calculated for the age range 15 to 49 as only a very small proportion of births occur to women outside of that age range. **Age-specific fertility rates** are usually calculated for single years of age or for 5-year age categories.

The concept of specific fertility arises out of the fact that fertility is affected by a number of factors such as age, marriage, state or region, urban-rural characteristics, etc. When fertility rate is calculated on the basis of age distribution, it is called the age-specific fertility rate. While calculating age-specific fertility rate women of different ages in the child-bearing age are placed in small age groups so as to put them at part with others of child-bearing capacity. The fertility of women differs from age to age and, therefore, the grouping of women of different ages is essential. The capacity to bear children is much higher in the age-group 20 to 25 than in the age-group 40 to 45.

$$\text{S.F.R.} = \frac{\begin{array}{c}\text{Number of live births which occured to female}\\ \text{of a specified age}-\text{group of the population of a}\\ \text{given geogrpahic area during a given year}\end{array}}{\begin{array}{c}\text{Mid}-\text{year female population of the specified}\\ \text{age}-\text{group in the given geographic area during}\\ \text{the same year}\end{array}} X\ 1000$$

Total Fertility Rate

Another frequently-used indicator is the total fertility rate, the average number of children born to a woman during her lifetime. The total fertility rate is generally a better indicator of current fertility rates because unlike the crude birth rate, it is not affected by the age distribution of the population. Fertility rates tend to be higher in less economically-developed countries and lower in more economically-developed countries. The TFR (or TPFR—total period fertility rate) is a better index of fertility than the Crude birth rate (annual number of births per thousand population) because it is independent of the age structure of the population, but it is a poorer estimate of actual completed family size than the total cohort fertility rate, which is obtained by summing the age-specific fertility rates that actually applied to each cohort as they aged through time. In particular, the TFR does not necessarily predict how many children young women now will eventually have, as their fertility rates in years to come may change from those of older women now. However, the TFR is a reasonable summary of current fertility levels. The total fertility rate (TFR) are sometimes also called the fertility rate, period total fertility rate (PTFR) or total period fertility rate (TPFR) of a population is the average number of children that would be born to a woman over her lifetime if:

1. she were to experience the exact current age-specific fertility rates (ASFRs) through her lifetime, and
2. she were to survive from birth through the end of her reproductive life. It is obtained by summing the single-year age-specific rates at a given time.

The TFR is a synthetic rate, not based on the fertility of any real group of women since this would involve waiting until they had completed childbearing. Nor is it based on counting up the total number of children actually born over their lifetime. Instead, the TFR is based on the age-specific fertility rates of women in their "child-bearing years," which in conventional international statistical usage is ages 15–44 or 15-49. Replacement fertility is the total fertility rate at which newborn girls would have an average of exactly one daughter over their lifetimes. That is, women have just enough female babies to replace themselves (or, equivalently, adults have just enough total babies to replace themselves). If there were no mortality in the female population until the end of the childbearing years (generally taken as 44 or 49, though some exceptions exist) then the replacement level of TFR would be very close to 2.0 (actually slightly higher because of the excess of boy over girl births in human populations). However, the replacement level is also affected by mortality, asexuality, genetic disorders inhibiting procreation, and by women without the desire to have children. The replacement fertility rate is roughly 2.1 births per woman for most industrialized countries (2.075 in the UK for example), but ranges from 2.5 to 3.3 in developing countries because of higher mortality rates. Taken globally, the total fertility rate at replacement is 2.33 children per woman. At this rate, global population growth would trend towards zero.

The TFR is, therefore, a measure of the fertility of an imaginary woman who passes through her reproductive life subject to *all* the age-specific fertility rates for ages 15–49 that were recorded for a given population in a given year. The TFR represents the average number of children a woman would have were she to fast-forward through all her childbearing years in a single year, under all the age-specific fertility rates for that year. In other words, this rate is the number of children a woman would have if she was subject to prevailing fertility rates at all ages from a single given year, and survives throughout all her childbearing years.

In order to measure correctly the population growth, we calculate the number of children born per thousand females in the child-bearing age divided into different age-groups. This leads to the total fertility rate which is calculated by adding up the specific fertility rates belonging to different age-groups. The total fertility rate is the mean number of children which a female aged 15 can expect to bear if she lives until at least the age of 50. Provided she is subject to the given fertility conditions over the whole of her child-bearing period. The total fertility rate for a particular area during a given period is a summary measure of the fertility conditions operating in that area during that period. In order to calculate the total fertility rate we shall have to calculate specific fertility rates and then add them. Total fertility rate is thus the sum of the age-specific fertility rates from a given age to the last point of child-bearing age of a female. In practice, we can shorten this procedure by working in quinquennial age groups.

Specific Fertility Rate = $\frac{\text{Annual births to females aged x and under (x+5)}}{\text{Mean number of females aged x and under (x+5)}} \times 1000$

Such a specific fertility rate is the rate per 1,000 per annum at which the females in the particular age-group produce offspring.

If we add the quinquennial specific fertility rates and multiplies by 5, we shall have the total number of children which 1.000 females aged 15 will bear over their lifetimes. A calculation based on quinquennial age-groups involves only one-fifth of the arithmetic of one based on single age groups and is very nearly as accurate, Symbolically.

TFR = ΣSFR × t

Where t = the magnitude of the age class.

Limitations of Fertility Rates

A few limitations of the Fertility rates are to be mentioned here. Fertility rates do not give an idea of the rate of population growth as it only refers to the number of children. If majority of births are of male children, then the female population will be reduced so that fertility rate does not reveal correct position of population growth.

<u>REPRODUCTION RATES</u>

The fertility rates are unsuitable for giving an idea of the rate of population growth because they ignore the sex of the newly born children and their mortality. If the majority of births are those of boys the population is bound to decrease while the reverse will be the case if the majority of births are girls. Similarly, if mortality is ignored a correct idea of the rate of growth of population cannot be formed because it is possible a number of female children may die before reaching the child-bearing age. For measuring the rate of growth of population we calculate the reproduction rates. Reproduction rates are of two types:

1. Gross Reproduction Rate, and
2. Net Reproduction Rate.

Gross Reproduction Rate

The **gross reproduction rate** (**GRR**) is the average number of daughters that would be born to a woman (or a group of women) if she survived at least to the age of 45 and conformed to the age-specific fertility rate of a given year. It is often regarded as the extent to which the generation of daughters replaces the preceding generation of females. The GRR is particularly relevant where sex ratios are significantly affected by the use of reproductive technologies. Gross reproduction rate measures the rate at which a new born female would, on an average, add to the total female population, if they remained alive and experienced the age-specific fertility rate till the end of the child-bearing period. It is the sum of fertility rate till the end of the child-bearing period. It is the sum of age-specific fertility rates calculated from female births for each single year of age. It shows the rate at which mothers would be replaced by daughters and the old generation by the new if no mother died or migrated before reaching the upper limits of the child-bearing age, i.e., 49 years. Another underlying assumption is that the same fertility rate continued to be in operation. If the gross reproduction rate of a population is exactly 1. It indicates that the sex under consideration is exactly replacing itself; if it is less than I, the population would decline, no matter how the death rate may be and if it is more than 1. The population would increase, no matter how low the death rate may be. The gross reproduction rate is computed by the following formula:

$$\text{G.R.R.} = \frac{\text{Number of female births}}{\text{Total number of births}} \text{ X Total Fertility Rate}$$

Also $$\text{G.R.R.} = \frac{\text{Number of female children born to 1,000 women}}{1{,}000}$$

The G.G.R. is used as a measure of the fertility in a population. It is useful for comparing fertility in different areas or in the same area at different time periods. The G.R.R. could in theory range from 0 to about 5. Gross reproduction rate has an advantage over the total fertility rate because in its computation we take into account only the female babies who are the future mothers whereas in the total fertility rate we include both male and female babies that are born.

An important limitation of the gross reproduction rate is that it ignores the current mortality. All the girls born do not survive till they reach the child –bearing age. Hence the gross reproduction rate is misleading in that it inflates the number of potential mothers. This defect is removed by computing the net reproduction rate. The accuracy of gross reproduction rate depends on the accuracy with which age-specific fertility rates can be computed. The principal sources of error are: (1) under-registration of births, (2) mis-statements or inadequate statements of the age of mother at registration, and (3) errors in enumeration or estimation of the female population by age-groups.

SURVIVAL FACTOR

Survival factor indicates the number of females surviving in a particular age group. For example, if the survival factor is 956, it means that out of 1000 live born females for that age, 956 only survive. Survival factor = (No. of female children born to thousand women × No. of survivals out of thousand female children) ÷ 1000.

Net Reproduction Rate

An alternative fertility measure is the net reproduction rate (NRR). Gross reproduction rate adjusted for the effects of mortality is called the net reproduction rate. It measures the number of daughters a woman would have in her lifetime if she were subject to prevailing age-specific fertility and mortality rates in the given year. Thus, the net reproduction rate (NRR) is the average number of daughters that would be born to a female (or a group of females) if she passed through her lifetime conforming to the age-specific fertility and mortality rates of a given year. This rate is similar to the gross reproduction rate but takes into account that some females will die before completing their childbearing years. An NRR of one means that each generation of mothers is having exactly enough daughters to replace themselves in the population. The NRR is particularly relevant where sex ratios at birth are significantly affected by the use of reproductive technologies, or where life expectancy is low.

When the NRR is exactly one, then each generation of women is exactly reproducing itself. The NRR is less widely used than the TFR, and the United Nations stopped reporting NRR data for member nations after 1998. But the NRR is particularly relevant where the number of male babies born is very high – see gender imbalance and sex selection. This is a significant factor in world population, due to the high level of gender imbalance in the very populous nations of China and India. The gross reproduction rate (GRR), is the same as the NRR, except that - like the TFR - it ignores life expectancy. Though gross reproduction rate gives an idea about the growth of population, it excludes the effect of the mortality on the birth rate. The rte estimates the average number of daughters that would be produced by women throughout their lifetime if they were exposed at each age to the fertility and mortality rates on which the calculation is based. It thus indicates the rate at which the number of female births would eventually grow per generation if the same fertility and mortality rates remained in operation. A net reproduction rate of 1 indicates that on the basis of the current fertility and female mortality, the present female generation is exactly maintaining itself. Both fertility and mortality are taken into account while calculating net reproduction rate. In its calculation it is assumed that 1,000 mothers give birth to a certain number of girls of whom a percentage dies in infancy and certain percentage does not marry. Of married girls some would become widow and it is only the balance that passes through fertility period and adds to the population growth. The N.R.R. measures the rate at which female population is replacing itself. Thus, the net reproduction represents the rate of replenishment of that population.

The net reproduction rate is obtained by multiplying the female specific fertility rate of each age by the population of female survivors to that age in a life table and adding up the products. An allowance is thus made for mortality.

$$\text{N.R.R} = \frac{\Sigma\ (\textbf{Nop.female births X Survival rate})}{\textbf{100}}$$

In other words NRR is obtained by dividing number of female births to thousand newly born females on the basis of current fertility and mortality rates by 1000.

$$\text{Net Reproduction Rate} = \frac{\text{Number of female births to thousand newly born females on the basis of current Fertility and mortality rates}}{1000}$$

The Net reproduction rate is always less than Gross reproduction rate. Both the rates will be equal when all the newly born daughters reached the child bearing age and passed through it. The net reproduction rate in theory can range from 0 to 5. If the net reproduction rate is one, it indicates that on the basis of current fertility and mortality rates, a group of newly born females will exactly replace itself in the new generation. This means that the population will be constant. If the net reproduction rates are below one, it indicates a declining population and if it is more than one, the population has a tendency to increase.

If NRR = 1, the female population remains constant

If NRR< 1, the female population is declining

If NRR> 1, the female population is rising

MEASUREMENT OF MORTALITY

Mortality Rates (Death Rates)

Mortality rate

Mortality rate is a measure of the number of deaths (in general, or due to a specific cause) in a population, scaled to the size of that population, per unit of time. Mortality rate is typically expressed in units of deaths per 1000 individuals per year; thus, a mortality rate of 9.5 (out of 1000) in a population of 100,000 would mean 950 deaths per year in that entire population. It is distinct from morbidity rate, which refers to the number of individuals in poor health during a given time period (the prevalence rate) or the number of newly appearing cases of the disease per unit of time (incidence rate). The term "mortality" is also sometimes inappropriately used to refer to the number of deaths among a set of diagnosed hospital cases for a disease or injury, rather than for the general population of a country or ethnic group. This disease mortality statistic is more precisely referred to as "case fatality rate" (CFR).

The mortality conditions of population are studies by measuring the crude death rate, the specific death rate, standardised death rate and infant mortality rate..

The Crude Death Rate

The crude death rate is the number of deaths from all causes in a given period (year) per 1000 of population, in a given community (locality). The crude death rate for a given year tells us at what rate deaths have depleted the population over the course of the year. We can calculate the crude death rate for males and females separately. The crude death rate usually lies between 8 and 30 per 1,000. The female rate is generally lower than the male rate. In most countries, crude death rates have fallen substantially over the years. The annual crude death rate is defined as:

$$\text{Crude Death Rate} = \frac{\text{Annual Deaths}}{\text{Annual Mean Population}} \times 1{,}000$$

The level of the crude death rate is determined by:

i) The sex and age distribution of population; and
ii) The mortality of the population. i.e., average longevity of the population.

An old population can exhibit a relatively high crude death rate even if longevity is high (i.e. mortality is low). The crude death rate, which measures the decrease in a population due to deaths, is perhaps the most widely used of any vital statistics rate. This is so for two reasons:

1) It is relatively easy to compute
2) It has value as an index in numerous demographics and public health problems.

However, death-rate so computed is likely to be misleading especially when it is required to compare the death rates in two areas or in two occupations. It is because of the fact that mortality varies with sex age whereas the crude death rate marks all age differentials. It assumes that age-sex structures of the populations being compared are the same. However, in practice it is not so. Population composed of a high proportion of persons at the older ages where mortality is higher will naturally show a higher crude death rate than younger population.

The crude death rate may be used for comparing the mortality situations of the same place at different times, provided the periods compared are not too far apart, because in a stable, large community the age and sex compositions of the population change very slowly. If the time trend is studied for a long period of years the effect of population changes must be examined. Greater caution is necessary for comparison between areas, since rather significant differences in crude death rates may arise entirely from differences in the age-sex distribution of the populations. However, where it is known that the population distributions are approximately similar, or where the crude rate differences are large, as in any international comparisons, the crude rate has great value as an index of mortality.

Specific Death Rates

By themselves, crude death rates are not enough for a detailed study of the mortality conditions in as community. We often need to know more about deaths occurring in different section of the population. For instance, people interested in infant or child welfare work study death taking place under 1 year of age or in such age groups as 1-4 years, 5-9 years, etc. Those interested in maternal health, study how many deaths occurred among women of child-bearing age. Insurance companies are interested in deaths occurring at different ages of the population.

The formula for computing specific death rate is:

Annual death rate Specific for age =

$$\frac{\begin{array}{c}Number\ of\ deaths\ which\ occured\ among\ a\\ Specific\ age\ group\ of\ the\ popualtion\ of\ a\\ Given\ geographic\ area\ during\ a\ given\ year\end{array}}{\begin{array}{c}Mid-year\ population\ of\ the\ specified\ age\\ Group\ in\ the\ given\ geographic\ area\\ during\ the\ same\ yer\end{array}} X\ 1{,}000$$

These rates measure the risk of dying in each of the age groups selected for the computation. Usually such rates are computed for the entire span of years, and are further specified bys sex, so that rates specific for age and sex are available. The specificity by age and sex eliminates the differences which would be due to variation in population composition in respect of these characteristics, and to this extent, such rtes can be compared from one geographic area to another and from one time period to another. However, it does not eliminate other variables which also may be important, such as "occupation", "Literacy", and the like. Nevertheless, for general analytical purposes, the death rate specific for age and sex is one of the most important and widely applicable types of death rates. It also supplies one of the essential components required for computation of net reproduction rate and life tables.

Standardized Death Rates

The criticism of the crude death rte is that while making inter-area comparisons, it fails to take account of differences in the age (or age-sex) structure of the population in question and, thus, fails to reveal the "real" mortality. It ha been suggested, therefore, that the crude rates, be "adjusted" to allow for the known differences in the age composition of the population involved. Several methods have been proposed and different names have been applied to the results, some workers have called these hypothetical indices "adjusted rates". Others "standardized rates" till other "corrected rates". Perhaps, the most appropriate term is "adjusted rates", used with a prefix to identify the basis of the adjustment as, for example. "Age- adjusted death rate, and so forth.

The standardized death rate, abbreviated as SDR, is the death rate of a population adjusted to a standard age distribution. It is calculated as a weighted average of the age-specific death rates of a given population; the weights are the age distribution of that population. As most causes of death vary significantly with people's age and sex, the use of standardized death rates improves comparability over time and between countries. The reason is that death rates can be measured independently of the age structure of populations in different times and countries (sex ratios usually are more stable).

There are two principal methods of age adjustment: (1) the direct method; and (2) the indirect method.

Direct Method

Direct method gives the death rate that would occur in some standard population if it had the mortality of the given community, or death rate that would occur in the community if its population were distributed as that of the standard. The direct method of adjusting for age consists of weighting the specific rates not by the population of the area to which they refer as its implied in the computation of the crude rate, but by the population distribution of another area, chosen as a standard. In the direct method, the rates specific for one geographic area are multiplied by the corresponding populations of another area which, for this purpose, is considered as a "standard". The resulting expected number for each age group is summed, and the total is divided by the total standard population to obtain an age-adjusted rate".

The obvious defect of the direct method as a means of adjusting for the age differences of several populations is that it entails the choice of a "standard" population. The choice of this "standard" will naturally affect the magnitude of resulting adjusted rates and may change their relative positions with respect to each other. However, in eliminating bias on a national basis, it is customary to use the total population of the country as "standard" for adjusting the rates of the regions within the country. There is no generally accepted standard population for international comparisons.

Indirect Method

The indirect method adjusts the crude rate of the community by applying to it a factor measuring the relative "mortality proneness" of the population of the community. In this method the "standard" is a set of specific rates, rates than a population distributed by age. To compute an age-adjusted rate by indirect method, one requires the population of the area distributed by age. These given populations are multiplied age by age by the "standard" age specific rates to obtain the expected number of events in the standard area if it were subject to the given age distribution. The sum of these "expected events" divided by the population in the area under consideration gives an "expected rate" or "index rate" in the "standard area" – one which is dependent solely on the sex-age constitution of the population and as a rule may be treated with sufficient accuracy as remaining constant over a period of years adjacent to experience period. The "index rate" is customarily divided into the crude rate of the "standard" area and the resulting ratio is known as an "adjustment factor" which can be used to adjust the crude rate of the area under consideration. Both the direct and indirect methods of age adjustments have been criticized on the ground that the rates obtained are dependent on the age and sex structure of the standard population used and that greater gains in mortality reduction obtained at younger age are not adequately accounted for.

As method for standardization, the direct method is applied. Standardized death rates are calculated for the age group 0-64 ('premature death'), 65 years and more and for the total of ages. As most causes of death vary significantly with people's age and sex, the use of standardized death rates improves comparability over time and between countries

Illustration 1: Compute the crude and standardized death rates of the two populations A and

B from the following data:

Age-group (years)	A		B	
	Population	Deaths	Population	Deaths
Below 5	15,000	360	40000	1000
5 - 30	20,000	400	52000	1040
Above 30	10,000	280	8000	240
Total	45,000	1,040	1,00,000	2,280

Solution.

Crude Death Rate = $\frac{N}{P}$ x 1000, where N = No. of deaths, P = Population

C.D.R. for town A = $\frac{1,040}{45,000}$ x 1,000= 23.11

C.D.R. for town B = $\frac{2,280}{21,00,000}$ x 1,000 = 22.80

Standardised Death rate, taking population of town A as standard population:

Age-Group (Years)	A			B		
	Population	Deaths	Death Rate per Thousand	Population	Deaths	Death Rate per Thousand
Below 5	15,000	360	24	40,000	1,000	25
5 -30	20,000	400	20	52,000	1,040	20
Above 30	10,000	280	28	8,000	240	30
Total	45,000	1,080		1,00,000	2,280	

Standardized Death Rate (town A)

$$= \frac{(15000\ x\ 24)+(20000\ x\ 20)+ (10000\ x\ 28)}{15000+20000+10000}$$

$$= \frac{360000+400000+280000}{45000} = \frac{1040000}{45000} = 23.11$$

Standardized Death Rate (town B)

$$= \frac{(15000\ x\ 25)+(20000\ x\ 20)+ (10000\ x\ 30)}{15000+20000+10000}$$

$$= \frac{375000+400000+300000}{45000} = \frac{1075000}{45000} = 23.89$$

We can now say that the death rate in town B is higher than in town A.

Another method of computing standardized death rate is to take some assumed population (that is the population of neither town A nor B) as standard. But this method is not popular.

Illustration 2: From the following table compare the death rates in two towns A and B. Which town is healthier?

Age-group	Town A		Town B		Standard Population	
	Population	No of Deaths	Population	No of Deaths	Population	No. of Deaths
0-10	4000	36	3000	30	2000	60
20-25	12,000	48	20000	100	8000	8
25-60	6000	60	4000	48	6000	4
60 and over	8000	152	3000	60	4000	50

Solution.

COMPARING DEATH RATES IN TOWNS A and B

Age-Group	Town A			Town B			Standard Population		
	Popu-lation	Deat hs	Death Rate	Populat-ion	Deaths	D.R.	Populat-ion	Deaths	D.R .
Below10	4000	36	9	3000	30	10	2000	60	30
10-25	12000	48	4	20000	100	5	8000	8	1
25-60	6000	60	10	4000	48	12	6000	4	0.67
60 and Above	8000	152	19	3000	60	20	4000	50	12.5

Standardized death rate (Town A)

$$= \frac{(2000x\ 9)+(8000x\ 4)+\ (6000x\ 10)+\ (4000x\ 9)}{20000}$$

$$= \frac{18000+32000+60000+76000}{20000} = \frac{186000}{20000} = 9.3$$

Standardized Death Rate (town B)

$$= \frac{(2000x\ 10)+(8000x\ 5)+\ (6000x\ 12)+\ (4000x\ 20)}{20000}$$

$$= \frac{20000+400000+72000+80000}{20000} = \frac{212000}{20000} = 10.69$$

Death rate is lower in town A as compared to B, hence town A is healthier. Since the S.D.R. of town A is less than that of town B, hence town A is more healthy.

Distinction between Crude Death Rate and Standardised Death Rate

1. Crude Death Rate is used as a measure of mortality of a population where asStandardised Death Rate is used for comparison of mortality of two populations.

2. Crude Death Rate requires limited information like number of deaths and total population. Standardised Death Rate requires more information. It requires a standard population.

3. Crude Death Rate does not take into account the age composition of the population while Standardised Death Rate requires it.

Age Specific Mortality Rates

The death rate calculated for a specified segment of population whose age is ‘n’ years in the last birth day is called age specific mortality rate. Different kinds of age specific mortality rates are :

Infant Mortality Rate

The most widely used definition of Infant mortality rate (IMR) is the number of deaths of babies under one year of age per 1,000 live births. The rate in a given region, therefore, is the total number of newborns dying under one year of age divided by the total number of live births during the year, then all multiplied by 1,000. The infant mortality rate is also called the infant death rate (per 1,000 live births). Infant mortality rates serve as one of the best indices to the general “healthiness” of a society. It is similar to age specific death rate for infants under 1 year of age. It is defined as:

$$\text{Infant Mortality Rate} = \frac{\begin{array}{c}\text{Number of deaths under 1 year of age which}\\ \text{occured among the population of a given}\\ \text{geographic area during a given year}\end{array}}{\begin{array}{c}\text{Number of live births which occurred among}\\ \text{the population of the given geographic area'}\\ \text{during the same year}\end{array}} \times 1000$$

The rate approximately measures for a given year the chances of a birth failing to survive one year of life. Still births are not included in the infant deaths. The rate can be calculated for males and females separately. The infant mortality rate varies considerably according to time and place. In countries with high standards of maternal and infant welfare it is as low as 15 to 20 per 1,000 but in some underdeveloped countries it is still well over 100 per 1,000. In many countries it has fallen spectacularly over the past sixty years or so. The male rate is appreciably higher than the female rate.

The infant mortality rate is of great value in the field of public health and its correct computation and interpretation is important. In most countries, the great risk of death at age under 1 is not equaled again in the life span until very old age is reached. But in contrast to deaths at old age, infant deaths are more responsive to improvement in environmental and medical conditions.

Neo-Natal Mortality Rate

The neo-natal mortality rate, like the infant mortality rate, is similar to an age specific rate. It is a rate used to measure the risk of death during the first month of lie. This rate is defined as:

$$\text{Annual Neo-natal Mortality rate} = \frac{\begin{array}{c}\text{Annual deaths of inants under the}\\ \text{age of 1 month among the population of}\\ \text{given geographic area}\end{array}}{\begin{array}{c}\text{Number of live births which occured among}\\ \text{population of a given geographi}\\ \text{area during the same year}\end{array}}$$

The rate measures for a given year the chance of a birth failure to survive one month of life. Most infant deaths occur within the first month of birth. The neo-natal mortality rate represents to a very large extent hard core of infant mortality. Of the neo-natal, deaths more occur within the first month of birth month of birth. The neo-natal mortality rate represents to a very large extent hard core of infant mortality. Of the neo-natal, deaths more occur within the first week of life.

Interpreting neo-natal rates, care must be taken to evaluate the probable effect of under-registration of live births in relation to infant deaths. It is likely that infant deaths, under 1 month, are registered less completely than any other infant deaths and the two sources of incompetencies in the rate probably compensate for each other to some extent.

Maternal Mortality Rate

The risk of dying from causes associated with child-birth is measured by the maternal mortality rate. For this purpose the deaths used in the numerator are those arising from puerperal causes. I.e. deliveries and complications of pregnancy, child-birth and the puerperium. The numbers exposed to the risk of dying from puerperal causes are women have been pregnant during the period. Their number being unknown the number of live births is used as the conventional base for computing comparable maternal mortality rates. The formula is:

$$\text{Annual maternal mortality} = \frac{\begin{array}{c}\text{Number of deaths from puerperal causes}\\ \text{occured among the female population of a}\\ \text{given geographic area during a given year}\end{array}}{\begin{array}{c}\text{Number of live births which occured}\\ \text{among the population of the given}\\ \text{geographic area during the same year}\end{array}}$$

The classification and coding of deaths as puerperal deaths vary from one country to another or even within the same country and hence we must be cautious in comparing maternal mortality rates for different places.

GROWTH OF POPULATION

Crude rate of natural increase of population

Crude Rate of natural increase of population in a country is the net result of difference between birth and deaths. Crude rate of natural increase = birth rate – death rate. Birth rates and death rates are expressed in 1000.

Intercensus Estimation of Population (Estimation of Population between Two Censuses)

The population is not counted every year but only once in 10 years. But it becomes necessary to know population for a particular year, known as mid-year estimated population. These estimates of the population are made from the preceding census figures. The three important methods of estimating population for the inter-censal and post censal years are i. Natural increase method, Arithmetic Progression Method and Geometric Method or Compound Interest method.

Natural increase method

Population in a year ‘1’ = Population in the year ‘0’ + (Births – Deaths) + (Immigration – Emigration) in the year 0.

$$= P0 + (B - D) + (I - E)$$

Arithmetic Progression Method

It is assumed that the population increase in A.P year by year by comparing the figures of any two census years. That is the increase in the population is assumed to be constant. The formula for estimating population for any day is given by $Pe = P1 = \frac{n}{N}(P2 - P1)$

Where Pe required population estimate, P1 and P2 are population determined in two census, preceding and succeeding, N = Number of years between two census, n = number of years between the dates of P1 and Pe.

Geometric Method (Compound Interest method)

If we assume the rate of growth of population year by year, as constant, instead of the increase between two census year populations to be constant, the procedure is known as Geometric Progression Method of estimating population. So in G.P method, we assume a constant rate of increase of population over the previous year. Population at the n^{th} year is

$P_n = P_0 \left(1+\frac{r}{100}\right)^n$ where P_0 = Population at year ‘n’.

P_0 = initial population

r = rate of growth of population

n = number of years

Sex Ratio

Sex ratio is the ratio of males to females in a population. This is defined as the total number of females living out of 1000 males living in a specified locality or region.

$\therefore$ Sex Ratio = $\frac{Total\ Number\ of\ Females}{Total\ Number\ of\ Males}$ x 1000

According to 2001 census, the sex ratio in Kerala is 1,058: 1 and at national level it is 0.933:1. That means, in Kerala, out of 1000 males the number of females is 1058. But at national level, there are only 933 females out of 1000 males.

Couple Protection Ratio

The couple protection rate (CPR) is usually expressed as the percentage of women in the age group of 15-49 years, protected from pregnancy/child birth in the year under consideration for a specific area. So When the Couple Protection Rate is going up Birth rate must necessarily fall. Among the important factors influencing CPR are available methods of Birth Control, distribution of supply, service and follow-up centres and their staffing patterns. Couple Protection Ratio (CPR) or Couple Protection Rate is usually expressed as the percentage of women in the age group 15-49 years protected from pregnancy or child birth in the year under consideration for specific area. CPR is 72 % in Kerala. At the national level the CPR is 52 %as per the Health Development Indicators for the year 2007.

Solved Problems

Example 1.The Population of a town in Karnataka as per 1981 census was 37.043 millions with birth rate and death rate of 34.3 and 12.7 respectively. What were the total number of births and deaths for 1981?

Ans: Given B.R = $\frac{B}{P}$ x 1000 ie 34.3 = $\frac{B}{37043000}$ x 1000

Therefore B = 34.3 x 37043 = 12,70,575

Therefore number of births = 1270575.

Given, DR = $\frac{D}{P}$ x 1000 ie 12.7 = $\frac{D}{37043000}$ x 1000

Therefore, D = (12.7) (37043) = 4,70,446

Therefore number of deaths = 470446.

Example 2. Given the following data, estimate the population of the locality at the end of 1982.

i.	Population at the end of 1981	:	3,42,14,320
ii.	Number of Births in 1982	:	82,000
iii.	Number of Deaths in 1982	:	15,000
iv.	Number of immigrants in 1982	:	23,000
v.	Number of Emigrants in 1982	:	18,000

Solution:

i. Here, P_0 = 3,42,14,320 B = 82,000 D = 15,000

vi. I = 23,000 E = 18,000

Therefore, the estimate of population at the end of 1982 is

$P_t = P_0 + (B - D) + (I - E)$

= 3, 42, 14,320 + (82,000 - 15,000) + (23,000 - 18,000)

= 3, 42, 86,320.

Example 3: The mid-year population of a city in year was 4, 80,500. If there were 10420 births and 6120 deaths in the year in the city, compute the Crude Birth Rate and Crude Death Rate.

Solution

The Crude Birth Rate is:

$$CBR = \frac{\text{Total Number of births occuring in the year}}{\text{Total population at the mid point of the year}} \times 1000$$

$$= \frac{10420}{480500} \times 1000 \quad = 21.69$$

The Crude Death Rate is:

$$CDR = \frac{\text{Number of live births occuring in the year}}{\text{Total population of the given locality}} \times 1000$$

at the mid point of the year

$$= \frac{6120}{480500} \times 1000 \quad = 12.74$$

Example 4: The following table gives the age and sex distribution and the number of live births in a population.

Age (Years)	0-89	19-20	20-24	25-29	30-39	40-59	60 and above
Male Population	13,470	10,342	9,210	7,912	5,915	4,343	6,433
Female Population	12,130	9,942	9,013	7,913	5,910	4,413	6,942
Number of live births to females	0	294	527	632	312	36	0

a. Find the Crude Birth Rate

b. Find the age Specific Fertility Rates for the age group 20-24 years and 20 - 39 years.

Solution

Age Group (Years)	Population		Number of births to Females
	Male	Female	
0-9	13,470	12,130	0
19-20	10,342	9,942	294
20-24	9,210	9,013	527
25-29	7,912	7,913	632
30-39	5 ,915	5,910	312
40-59	4,343	4,413	36
60 and above	6,433	6,942	0
Total	57,625	56,263	1,801

a) Here, total male and female populations are 57,625 and 56,263 respectively. Therefore, total population is 57,625 + 56,263 = 113,888. Total number of births is 1,801. Therefore, Crude Birth Rate is

$$\text{CBR} = \frac{\text{Number of live births occuring in the year}}{\text{Total population at the mid point of the year}} \times 1000$$

$$= \frac{1801}{113888} \times 1000 \quad = 15\ .81$$

b) Age specific Fertility Rate for the age group 20-24 years is

$$\text{ASRF} = \frac{\text{Number of live births in the year to females aged between 20 and 24}}{\text{Total of females aged between 20 and 24}} \times 1000$$

$$\frac{527 \times 1000}{9013} = 58.47$$

Age Specific Fertility Rate for the age group 20 -9 years is

$$\text{ASRF} = \frac{\text{Number of live births in the year to females aged between 20 and 30}}{\text{Total of females aged between 20 and 39}} \times 1000$$

$$= \frac{(527+632+312)}{9013+791+5910} \times 1000 = \frac{1471}{22836} \times 1000 = 64.42$$

Example 5 The population ofa village in Calicut on 1-7-2001 was 30,000. The vital events for the same year were;

The number of births: 1200, number of deaths: 495, number of infant death: 110. Compute the birth, death and infant mortality rates. Also compute the growth rate.

Ans: B = 1200, P = 30,000, D = 495, $D_I = 110$

Birth Rate = (B/P) x 1000 = $\frac{1200}{30000}$ x 1000 = 40 per thousand

Death Rate = (D/P) x 1000 = $\frac{495}{30000}$ x 1000 = 16.5 per thousand

Infant Mortality Rate = (D_I/B) x 1000 = $\frac{110}{1200}$ x 1000 = 91.66 per thousand

Growth Rate = $\frac{B.R - D.R}{1000}$ x 100 = $\frac{40-16.5}{1000}$ x 1000 = 2.35%

Example 6 Calculate Age Specific Death Rates.

Age (Years)	Population	Number of Deaths
Below 10	30,000	420
10-19	20,000	150
29-29	25,000	125
30-39	8,000	70
40-49	2,500	25
50 & Above	2,000	30

Solution

Here, the Age Specific Death Rates are found by using the formula –

$$\text{ASDR} = \frac{\text{Number of deaths occuring in the age group in the year}}{\text{Total population in the age group in the year}} \times 1000$$

Age	Population	Number of Deaths	$\frac{\text{Deaths}}{\text{Population}} \times 1000$
Below 10	30,000	420	$\frac{420}{30000} \times 1000 = 14$
10-19	20,000	150	$\frac{150}{20000} \times 1000 = 7.5$
20 -29	25,000	125	$\frac{125}{25000} \times 1000 = 5$
30-39	8,000	70	$\frac{70}{8000} \times 1000 = 8.75$
40-49	2,500	25	$\frac{25}{2500} \times 1000 = 10$
50 & Above	2,000	30	$\frac{30}{2000} \times 1000 = 15$

Example 7 Calculate Crude Death Rate and Standardised Death Rate from the following data.

Age (Years)	Population (Thousand)	No. of Deaths	Standard Population (Percentage Distribution)
0-9	21	350	22
10-24	30	102	30
25-44	37	229	28
45-64	17	54	15
65 & Above	5	415	5

Solution

Age	Population	Deaths	A = ASDR	Standard Population (P)	PA
0-9	21000	350	16.67	22	366.74
10-24	30000	102	3.40	30	102.00
25-44	37000	229	6.19	28	173.32
45-64	17000	54	20.82	15	312.30
65 & Above	5000	415	83.00	5	415.00
Total	110000	1450		100	1369.36